Earnings dynamics and optimal capital structure

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Abstract

We develop a dynamic trade-off model with mean-reversion in earnings and a growth option. The firm optimizes initial debt and optimally adjusts leverage during the financing of growth option investment. We provide predictions and managerial implications on the impact of long-term profitability, mean reversion speed, volatility of earnings and debt priority structure on firm value, the dynamics of leverage and credit spreads and regarding the optimal timing of growth option investment and default. Our model shows that the relation between current profitability and leverage follows a U-shape. The U-shape is driven by growth options and we show that it also holds for firms with non-stationary earnings. We estimate stochastic process parameters and provide empirical evidence on the relation of leverage with profitability and other common factors used in the literature providing a first comparison between mean reverting and non-stationary firms.

Keywords: trade-off model; mean-reversion; investments; leverage; real options

1. Introduction

Financial flexibility is important for firms (see Graham and Harvey, 2001 and Graham, 2022) and dynamic models which consider adjustments in leverage and real investments provide a vehicle for understanding firms' capital structure decisions. Indeed, Strebulaev (2007) concludes that a proper study of the evolution of capital structure requires a model that combines both dynamic capital structure decisions and real investment. While theoretical models combining capital structure and real investment decisions are well-developed (e.g., Hennessy and Whited (2005), Titman and Tsyplakov (2003), and Hackbarth and Mauer (2012)), a link between the nature of shocks (i.e., temporary or permanent) on firm's cash flows and corporate financial decisions is still a challenging task. In this paper we focus on earnings dynamics and revisit the empirical predictions of dynamic trade off models, as well as the profitability-leverage relationship puzzle.[1]

We develop a model where earnings follow an arithmetic mean-reverting (AMR) process and incorporate dynamic financing decisions and a growth option. Levendorskii (2005) uses the AMR process to study optimal exercise boundaries and pricing of perpetual American call and put options. Recently, Briest et al. (2022) have applied this process to study the investment behavior of power grid-stabilizing, flexibility-providing energy projects. None of these papers study capital structure decisions. Instead, following Hackbarth and Mauer (2012), in our model the firm has two debt issue decisions (initially and when it finances the exercise of the growth option), a priority structure decision (for the initial and subsequent debt issue), an investment decision (when to exercise the growth option), and two default decisions (before and after the exercise of the growth option). The literature studying the effect of mean reversion on the investment and disinvestment decisions of firms has used mostly the geometric mean-reverting (GMR) process (Sarkar, 2003; Tserkrekos, 2010; Metcalfe and Hassett, 1995, Reymar, 1991). The same type of process is employed in Sarkar and Zapatero (2003), where they reformulate Leland's (1994) trade-off model incorporating mean reversion in the corporate earnings process and study debt financing without investment decisions. With GMR, however, cash flows can never become negative and thus allowing for negative earnings might increase the importance of financial flexibility. In addition,

[1] We are further motivated by empirical studies that show that not properly considering earnings or cash flow dynamics may lead to misleading findings. One such example is leverage mean reversion (e.g., found in Fama and French, 2002 and Flannery and Rangan 2006). Chen and Zhao (2007) and Chang and Dasgupta (2009) also discuss hazards of not properly employing earnings dynamics.

a GMR process assumes volatility increases proportionally with profitability while in an AMR process volatility is independent of the profitability level. Transitory shocks have been used extensively to study optimal cash management policies (e.g., Décamps et al. 2016, Cadenillas et al, 2007). In a capital structure setting and more related to our framework, Gorbenko and Strebulaev (2010) provide a contingent claim trade-off model with both temporary and permanent shocks. In their model, the temporary component of the shocks is driven by Poisson jump shocks that arrive in discrete time and then fade in expectation over time so that earnings mean-revert to permanent levels. Raymar (1991) studies dynamic decisions within a mean reversion setting, however, he focuses on short-term (single period) debt and exogenous default. In comparison with earlier work, our analysis allows for analytic solutions involving a growth option, two stages of financing and endogenous default and thus provides a framework which can be contrasted with Hackbarth and Mauer (2012) to allow the study and comparison of predictions for both firms with non-stationary and stationary earnings processes.

One of our theoretical results challenges the traditional interpretation of the empirical negative relation between profitability and leverage, which is often cited against trade-off models and in favour of pecking order theory (see Shyam-Sunder and Myers (1999) and follow-up work). In contrast with Sarkar and Zapatero (2003), who find a negative relation between profitability and leverage, we find that leverage has a U-shape relationship with profitability.[2] This effect is driven by the presence of a growth option since at low levels of earnings equity increases at a faster rate due to the growth option upside potential, however, as the increase in current profitability brings the firm closer to exercising its growth option and enjoying an expansion of the net benefits of debt, debt value improves at a higher rate. Besides providing this new theoretical contribution in a mean reversion setting, we also contribute to the literature by clarifying that a U-shape (also growth option driven) relation exists for firms following non-stationary earnings, i.e., within our setting when mean reversion speeds tend to zero or within the Hackbarth and Mauer (2012) setting where earnings follow a GBM. These predictions are in stark contrast to predictions based on static trade-

[2] In the paper "profitability" refers to the level of earnings just like in Sarkar and Zapatero (2003). However, the U-shape relation holds also when associating rates of return (level of earnings scaled with assets) with leverage. The latter is more relevant for empirical work which associates rates of return with leverage. We thus expect that for firms with earnings following an AMR the relation between rates of return and leverage will generally follow a U-shape.

off models which predict a positive leverage-profitability relation. In Tserlukevich (2008) who uses the Geometric Brownian Motion assumption, inaction caused due to irreversibility and fixed costs creates a negative relation between leverage and profitability. In contrast, in our paper our focus is *at* investment points where we show that the leverage-profitability relation is generally U-shaped, irrespective of earnings dynamics. Danis et al. (2014) focus on dynamic inaction models where firms make infrequent capital structure adjustments along the theoretical lines of the model of Goldstein, Ju and Leland (2001). In contrast, our work focuses on investment related capital structure adjustments.

Furthermore, our theoretical analysis predicts that leverage is decreasing in earnings volatility and growth options (i.e., the expansion factor of future revenues) and positively related to the mean reversion speed and long-term profitability. We also provide predictions related to dynamic adjustments (changes in leverage) when firms reach a state where they exercise their investments with no more growth options available. Our framework thus contributes to related works focusing on investment dynamics (Hennessy and Whited, 2005 and Dudley, 2012). We also provide further insights relating the debt conservatism puzzle relating to earnings dynamics. We find that a lower speed of mean reversion implies a higher volatility and results in more conservative (low) leverage levels. Our prediction is in line with Gorbenko and Strebulaev (2010) who show that equity holders benefit disproportionately from positive shocks and thus demand ex ante compensation which reduces equity holders' desire to rely on debt. In addition, we show that the higher debt conservatism shown in Hackbarth and Mauer (2012) under "me-first" priority structure of debt (compared to equal priority) is preserved in the presence of mean reversion. DeAngelo et al. (2018) shows that firms may indeed preserve financial flexibility for future investments.

In addition to the empirical predictions, our framework provides several new managerial implications regarding optimal investment timing and default decisions in the presence of mean reversion. We show that optimal investment is delayed for firms with earnings which are more volatile and have low levels of long-term profitability or firms with low levels of expansion (growth) options. The impact of mean reversion hinges upon the level of long-term profitability. When long-term profitability is high, an elevated degree of mean reversion accelerates investment. On the other hand, investment is postponed when profits mean-revert faster to low long-term profitability levels. Optimal default is delayed for firms with higher earnings volatility, higher levels of growth options and is U-shaped with respect to long-term profitability and mean reversion speed. Finally, we discuss how investment and default decisions are affected by the priority rules of debt at default.

In the empirical part of the paper, we first show how to estimate the parameters of the earnings process and using an Augmented-Dickey Fuller test document that this type of process characterizes a vast majority (about 60%) of the universe of non-financial, non-regulated US firms in the COMPUSTAT database with 40 consecutive quarterly earnings observations (that allows for reasonable accuracy in the estimation of model parameters). We provide the first evidence on the characteristics of both mean reverting and non-stationary firms. Our median mean reverting firm has more leverage, lower profitability, smaller size based on sales, less growth potential based on market to book and more tangible assets compared to our sample of non-stationary firms. The evidence appears consistent with studies that do not separate the two groups and do not place requirements of estimating the stochastic process parameters (e.g., Danis et al., 2014). The existence of more tangible assets for mean reverting firms is consistent with a larger concentration of mean reversion among manufacturing firms (about 75% compared to 51% for non-stationary firms). Our multivariate analysis confirms a negative relation of leverage with our estimated volatility measure and a positive relation with long-term profitability and mean reversion speed. However, we document a consistently negative relation of profitability with leverage both unconditionally, as well as conditional on investment events involving debt financing for both mean reverting and non-stationary firms. Our analysis provides further evidence that supports the leverage-profitability puzzle (see also Eckbo and Kisser, 2021) by accounting for earnings dynamics. We find however that the negative relation of profitability with leverage is reduced at high profitability levels for firms with growth options.

Our contributions can be summarized as follows. First, we extend earlier studies studying mean reversion in earnings by providing empirical predictions based on a more realistic mean reversion process, multiple financing stages and alternative priority rules for debt and a growth option. We also revisit and clarify the empirical predictions for non-stationary firms in the presence of growth options. Secondly, we provide the first evidence on the empirical prevalence of mean reversion and non-stationarity of earnings, a comparison of the characteristics of firms and the factors affecting leverage for the two groups of firms. Our study thus contributes to the literature attempting to better characterize firms' dynamic capital structure decisions with theoretically motivated predictions. This literature is extensive and includes work that relies on structural estimation (e.g., Hennessy and Whited, 2005 and Strebulaev, 2007) and work such as that of Danis et al. (2014) (see also Eckbo and Kisser, 2021) who test dynamic inaction models where firms make infrequent capital structure adjustments along the theoretical lines of Goldstein, Ju and Leland (2001). More related to our work, Tserlukevich (2008) focuses on a model with investments which incorporates irreversibility and fixed costs of investment, however, in comparison our paper shows new insights relating the leverage-profitability relation during investments events involving debt financing (and not in the inaction region) and clarifies the impact of earnings dynamics.

Our paper is organized as follows. Section 2 describes the theoretical model with mean reversion in earnings and a growth option. Section 3 presents the numerical sensitivity results and summarizes the model predictions as well as predictions for firms following non-stationary earnings. Section 4 shows the estimation approach of the earnings process and empirically applies this to the universe of US firms, while Section 5 concludes. Appendix 1 provides a summary of the notation of the theoretical model. An online Appendix provides details for the derivation of the theoretical model and additional tests.[3]

2. The model with mean reversion in earnings

2.1. Model assumptions

We model a firm with existing assets generating net cash flow or earnings x. The earnings stream x follows an arithmetic mean-reverting (ARM) process as follows:

$$dx = q(\theta - x)dt + \sigma dz \tag{1}$$

[3] Specifically, online Appendix 1 shows the derivation of the homogeneous differential equation solution, online Appendix 2 shows the derivation of the solutions for the basic and general claims involving two boundaries within a mean-reverting framework and online Appendix 3 shows the proofs for security and firm values presented in the main text. Online Appendix 4 presents additional sensitivity results of the main model and sensitivity that confirm the robustness of the U-shape of leverage with respect to profitability, irrespective of the earnings process, as well as results of sensitivity with respect to profitability in the absence of growth option.

where q defines the mean reversion speed, θ defines the long-term mean to which earnings revert, σ the project earnings volatility and dz is the increment to a standard Brownian Motion process. The firm has a growth opportunity to increase earnings to a level $e\ x$ at an optimal time. The firm selects an optimal level of perpetual debt $Db(x)$ at time zero (stage 1) with a promised (coupon) payment R_0 and pays corporate taxes at a constant rate τ with a full-loss offset scheme.[4]

The bankruptcy trigger x_b is endogenously and optimally chosen by equity holders by maximizing equity value. When earnings x drop to the low threshold level x_b then the firm goes bankrupt and the original debt holders take over and obtain the firm's unlevered assets $Ub(x)$ net of proportional bankruptcy costs b, $0<b<1$. On the other hand, if earnings rise to a high level x_I then the firm makes a capital (growth) investment I and expands earnings by $e > 1$, thus earnings after investment become $v = e\ x$. The optimal timing for investment is chosen to maximize the market value of equity ("second-best investment"). Post investment earnings also follow a mean-reverting process of the following form:

$$dv = q(e\theta - v)dt + e\sigma dz \tag{2}$$

Thus, after investment, earnings follow an AMR process with standard deviation $e\sigma$ and long-term mean $e\theta$.

New investment can be financed by additional perpetual debt $Da(x)$ with coupon R_1. Post investment, equity holders select the earnings level v_L which triggers bankruptcy. Priority rules define the amount of unlevered assets obtained by original and subsequent debt holders in the event of bankruptcy. Like Hackbarth and Mauer (2012) we allow for commonly observed priority rules which include absolute priority of original debt, pari-passu (equal priority) and absolute priority for subsequent debt holders.

The optimization of capital structure is performed by selecting the initial coupon R_0 and subsequent coupon level R_1 jointly with optimally chosen investment and default levels. R_0 is chosen to maximize initial firm value (equity plus initial debt financing obtained) while R_1 is chosen to maximize equity plus the proceeds from the new debt issue. This amounts to "second-best financing", as suggested in Hackbarth and Mauer (2012). We do not focus on agency

[4] We do not consider tax convexity issues (see Sarkar 2008) but assume that constant tax rate τ is applied irrespective of the earnings level. Our analysis thus likely exaggerates somehow the true tax benefits levels.

considerations in this paper and thus do not consider a comparison with a "first-best" optimization for either the selection of investment timing and/or financing. "First-best" investment timing would be the one that caters for debt holders value by maximizing firm (instead of equity only) value and "first-best financing" allows that the choice of R_1 caters for the dilution effects on initial debt (see Hackbarth and Mauer, 2012 for further details).

2.2. Security and firm valuation after investment

With an AMR process the solution to the value of claims becomes complex. In Online Appendix 1 we show the derivation of the solutions to the homogeneous differential equations with AMR. Below, we utilize the solution of the basic claims derived in Online Appendix 2. Online Appendix 3 provides full details relating the valuation of all claims.

2.1.1. Equity and unlevered assets after investment

Equity value after investment is equal to:

$$Ea(v) = Ea_p(v) - Ea_p(v_L)\left(\frac{P_1(v)}{P_1(v_L)}\right) \tag{3}$$

where $v = ex$ are expanded cash flows following investment and

$$Ea_p(v) = \left(\frac{1}{q+r}v + \frac{q\theta^*}{r(q+r)} - \frac{R_0+R_1}{r}\right)(1-\tau) \tag{4}$$

with $\theta^* = e\theta$.

In equation (3) the term $P_1(\cdot)$ is defined in equation (5a) below. Equation (5b) also defines $P_2(\cdot)$ that will be used in subsequent equations for the value of securities.

$$P_1(x) = e^{\frac{1}{4}\left(\frac{(x-\theta)\sqrt{2q}}{\sigma}\right)^2} D_v\left(\frac{(x-\theta)\sqrt{2q}}{\sigma}\right) \tag{5a}$$

$$P_2(x) = e^{\frac{1}{4}\left(\frac{(x-\theta)\sqrt{2q}}{\sigma}\right)^2} D_v\left(-\frac{(x-\theta)\sqrt{2q}}{\sigma}\right). \tag{5b}$$

where $D_v(z) = \frac{1}{2^{\xi}\sqrt{\pi}}\left[\cos(\xi\pi)\,\Gamma\left(\frac{1}{2}-\xi\right)\,y_1(a,z) - \sqrt{2}\sin(\xi\pi)\,\Gamma(1-\xi)\,y_2(a,z)\right]$ (6)

$$z = \frac{x-\theta}{\bar{\sigma}}, \bar{\sigma} = \sigma/\sqrt{2q}$$

$$a = -\nu - \frac{1}{2}, \nu = -\frac{r}{q} < 0$$

$$\xi = \frac{1}{2}a + \frac{1}{4}$$

$\Gamma(\cdot)$= is the Gamma function

$$y_1(a, z) = e^{-\frac{z^2}{4}} \ {}_1F_1 \ \left(\frac{1}{2}a + \frac{1}{4}; \frac{1}{2}; \frac{z^2}{2}\right)$$

$$y_2(a, z) = z\, e^{-\frac{z^2}{4}} \ {}_1F_1 \ \left(\frac{1}{2}a + \frac{3}{4}; \frac{3}{2}; \frac{z^2}{2}\right)$$

In the above ${}_1F_1 \ (\alpha; \beta; z) = M(\alpha; \beta; z)$ is the confluent hypergeometric function (see Abramowitz and Stegun, 1972). The Gamma function is defined as follows:

$$\Gamma(n) = \int_0^\infty x^{n-1} e^{-x} dx$$

where the integral converges for $n > 0$. Note that $\Gamma(n+1) = n\Gamma(n)$, so for integer n this function coincides with the factorial function, that is, $\Gamma(n+1) = n!$.

Note that in equation (3) the term $Q(v) = \frac{P_1(v)}{P_1(v_L)}$ can be interpreted as the value of a basic claim which pays one dollar when v_L is reached from above from v.

The value of unlevered assets after investment is:

$$Ua(v) = \left[\frac{1}{q+r} v + \frac{q\theta^*}{r(q+r)}\right](1 - \tau) \tag{7}$$

In expression (7) the term $\frac{1}{q+r}v$ represents the transitory component and the constant $\frac{q\theta^*}{r(q+r)}$ is a permanent component. Note that when $q = 0$, then expression (7) simplifies to $v(1-\tau)/r$, which is the value for an arithmetic process with zero drift. When the earnings level changes, the value $Ua(v)$ is affected only by the transitory part. Since the transitory part is a decreasing function of the speed of reversion q, if mean reversion becomes stronger (q increases), the transitory part becomes less important and if q goes to infinity, it disappears. To avoid negative liquidation values for initial debt holders at bankruptcy we ensure that $Ua(v)$, as well as $Ub(x)$, do not drop below zero at the bankruptcy thresholds (see Online Appendix 3 equations A29 and A44).

2.2.1. Debt and firm value after investment

Debt value after investment for the initial debt issued at time zero $Da_0(v)$ and the second debt issued at the investment trigger $Da_1(v)$ are given by:

$$Da_i(v) = \frac{R_i}{r} + \left(Da_i(v_L) - \frac{R_i}{r}\right)\left(\frac{P_1(v)}{P_1(v_L)}\right) \tag{8}$$

where $Da_i(v_L)$ depends on the priority structure. In the case of equal priority of the two debt issuers, liquidation proceeds are shared depending on the scale of payments:

$$\beta_0 = \frac{R_0}{R_0+R_1}, \qquad \beta_1 = 1 - \beta_0 = \frac{R_1}{R_0+R_1}$$

Thus, with equal priority the boundary condition for debt becomes:

$$Da_i(v_L) = \beta_i\,(1-b)\,Ua(v_L) \tag{9}$$

In the case the first lender has secured priority to other creditors ("me-first" for initial debt) then the boundary conditions become:

$$Da_0(v_L) = min\,\left[(1-b)\,Ua(v_L), \frac{R_0}{r}\right] \tag{10a}$$

$$Da_1(v_L) = (1-b)\,Ua(v_L) - Da_0(v_L) \tag{10b}$$

In the case that second lender have secured priority to other creditors then the boundary conditions become:

$$Da_1(v_L) = min\,\left[(1-b)\,Ua(v_L), \frac{R_1}{r}\right] \tag{11a}$$

$$Da_0(v_L) = (1-b)\,Ua(v_L) - Da_1(v_L) \tag{11b}$$

Firm value after investment is then given by the sum of equity plus debt values after investment:

$$Fa(v) = Ea(v) + Da_0(v) + Da_1(v) \tag{12a}$$

Replacing equation (3) and equations (8) for $Ea(v), Da_0(v)$ and $Da_1(v)$ in equation (12a) above we obtain an alternative characterization of firm value as follows:

$$Fa(v) = Ua(v) + TBa(v) - BCa(v) \tag{12b}$$

where $Ua(v)$ is given in equation (7) and $TBa(v), BCa(v)$ are defined as follows:

$TBa(v) = \left(\frac{R_0+R_1}{r}\right)\tau - \left(\frac{R_0+R_1}{r}\right)\tau\left(\frac{P_1(v)}{P_1(v_L)}\right)$, $BCa(v) = bUa(v_L)\left(\frac{P_1(v)}{P_1(v_L)}\right)$. We also define $NBa(x) = TBa(v) - BCa(v)$ as a summary measure of the net benefits of debt after investment.

2.3. Valuation before investment

2.3.1. Equity and unlevered value before investment

Equity value before investment $Eb(x)$ is given by:

$$Eb(x) = \left(Ea(e\,x_I) - I + Da_1(e\,x_I) - Eb_p(x_I)\right)J(x) - Eb_p(x_b)\,L(x) + Eb_p(x) \qquad (13)$$

where $Eb_p(x) = \left(\frac{1}{q+r}x + \frac{q\theta}{r(q+r)} - \frac{R_0}{r}\right)(1-\tau)$.

$J(x)$ in equation (13) defines the value of a basic claim that pays one dollar if x hits trigger x_I and zero when it hits x_b. Similarly, we define a basic claim $L(x)$ that pays one dollar if x hits trigger x_b and zero when it hits x_I. The solutions to these basic claims are as follows (see Online Appendix 1 and 2):

$$J(x) = \frac{P_2(x_b)}{D(x_I,x_b)}P_1(x) - \frac{P_1(x_b)}{D(x_I,x_b)}\,P_2(x) \qquad (14)$$

$$L(x) = -\frac{P_2(x_I)}{D(x_I,x_b)}P_1(x) + \frac{P_1(x_I)}{D(x_I,x_b)}\,P_2(x)$$

where $D(x_I, x_b) = P_1(x_I)P_2(x_b) - P_1(x_b)P_2(x_I)$.

The value of unlevered assets before investment is given by:

$$Ub(x) = \left[\frac{1}{q+r}x + \frac{q\theta}{r(q+r)}\right](1-\tau) \qquad (15)$$

2.3.2. Debt and firm value before investment

Initial (t = 0) debt value is given by:

$$Db(x) = \frac{R_0}{r} + \left(Da_0(x_I) - \frac{R_0}{r}\right) J(x) + \left((1-b)\; Ub(x_b) - \frac{R_0}{r}\right) L(x) \qquad (16)$$

where equation for $Da_0(x)$ is given in equation (8) and $Ub(x)$ in equation (15).

Thus, firm value before investment is the sum of equity plus debt before investment:

$$Fb(x) = Eb(x) + Db(x) \qquad (17a)$$

Replacing equation (13) for $Eb(x)$ and equation (16) for $Db(x)$ we obtain the following breakdown of firm value at t = 0:

$$Fb(x) = UB(x) + Ua(v_I)\, J(x) + TBb(x) + TBa(v_I)\, J(x) - BCb(x) - BCa(v_I)J(x) - I\, J(x)$$
(17b)

where $UB(x) = Ub(x) - Ub(x_I)J(x)$ with $Ub(\cdot)$ given in equation (15), $TBb(x) = \frac{\tau R_0}{r} - \frac{\tau R_0}{r} J(x)$-$\frac{\tau R_0}{r} L(x)$ and $BCb(x) = bUb(x_b)L(x)$. We also define the net benefits of debt at t = 0 as $NBb(x) = TBb(x) - BCb(x)$.

2.4. Optimal investment, default, and capital structure

In this section we describe smooth pasting (optimality) conditions. First, we demand that the derivative of equity after investment at v_L should be zero to ensure that equity holders choose the bankruptcy trigger optimally following investment. This implies the condition:

$$Ea'(v_L) = 0\,. \qquad (18)$$

Note that the optimality condition in equation (18) can be stated in terms of the underlying x with the condition $\tilde{E}a'(x_L) = 0$ where $\tilde{E}a(\cdot)$ is equation defined in equation (3) evaluated at $v = e\,x$. Similarly, we demand that the derivative of equity value before investment should be zero at bankruptcy trigger x_b:

$$Eb'(x_b) = 0. \qquad (19)$$

We use "second-best investment" optimization for the investment trigger x_I which accounts for raising the optimal new level of debt financing, however, it does not account for the effect of investment on existing debt holders. This translates into:

$$Eb'(x_I) = \tilde{E}a'(x_I) + \tilde{D}a_1{}'(x_I) \qquad (20)$$

Note that under "first-best investment" optimization (not used in our subsequent analysis) equity holders consider the best interest of debt issuers by optimizing firm value. This would imply the following condition $Fb'(x_I) = \tilde{F}a'(x_I)$ where $\tilde{F}a'(x_I)$ is equation (12) replacing $v = e\ x$. "First-best investment" would be useful for analysing agency issues which is not the goal of this paper.

The optimal capital structure is selected by performing a dense grid search for both the initial and subsequent coupon levels such that R_0 and R_1. R_0 is chosen to maximize initial firm value, i.e., equity plus initial debt financing obtained (see equation, 17a) while R_1 is chosen to maximize equity plus the proceeds from the new debt issue (equation 20). Both R_0 and R_1 also satisfy optimally chosen investment and default levels (see equations 18, 19). This optimization identifies the initial and subsequent debt levels in the firm's capital structure. Note that due to equation (20) the coupon level R_1 is chosen to maximize equity plus the new debt proceeds ("second-best financing").

3. Hypotheses development

3.1. Sensitivity analysis of the model with mean reversion in earnings

In this section we provide numerical sensitivity results with respect to main parameters that will be analyzed also empirically next which are the mean reversion speed (q), long-term profitability (θ), earnings level (x) and growth option expansion factor (e). We provide a summary of the effect of other parameters in Appendix A.4.1. We explore the impact of these variables on firm value, leverage ratio levels and the change in leverage ratios when growth options are exercised, and the credit spreads. We also study the effect of alternative priority rules for debt. We also focus on the development of testable empirical predictions relating to leverage ratios in the cross section. Before moving on, it is important to clarify that the sensitivity results relating the initial leverage ratios at $t = 0$ reported in our sensitivity analysis (see Lev_b reported in subsequent tables) provide a natural way to form predictions on the leverage in the cross section when firms exercise their investments by also considering future growth opportunities being available. This is despite the fact we do not impose an initial cost at t = 0 since imposing an initial cost would not alter the directional effects of comparative statics. On the other hand, the leverage ratios reported at the follow-on investment stage (Lev_T reported in subsequent tables) reflect leverage ratios when firms reach a state where future options are no longer available (e.g., firms reaching a mature steady state). We report these follow-on leverage ratios since they provide interesting time dynamics (e.g., firms leverage comparisons between young vs mature firms), however, our subsequent hypotheses and empirical tests rely solely on predictions relating the leverage ratios in the cross section based on initial leverage ratios when future growth opportunities exist.

Our base case parameters are as follows. We use a normalized level of current earnings at the level $x = 1$. Following Hackbarth and Mauer (2012) we take a risk-free rate of r = 0.06, a tax rate τ = 0.15. For the growth option we also follow the same study and use $e = 2$ and an investment cost $I = 10$ (i.e., the cost of investment is ten times the current earnings level as used in Hackbarth and Mauer, 2012). We take proportional bankruptcy costs $b = 0.5$ as in Leland (1994). For the mean-reverting stochastic process parameters we follow Sarkar and Zapatero (2003) and use σ = 0.4, mean reversion speed q = 0.1 and long-term mean $\theta = 1$. The parameters of the AMR process are in line with empirical estimates provided in the empirical section of the paper.

For all subsequently reported results we report sensitivity until $x < x_I$ remains valid. In all simulations we ensure that the value of unlevered assets at default thresholds which defines recovery values for debt holders at default (net of bankruptcy costs) is never negative (see discussion following equations A29 and A44 in the Online Appendix 3). Finally, where necessary to show a particular direction of an effect more clearly, we provide a denser sensitivity analysis.

Table 1 provides interesting new insights with respect to mean reversion speed q. We observe that an increase in mean reversion speed q decreases x_I which implies an acceleration of investment irrespective of long-term levels of profitability (see Panel A and also that $Inv_b(x)$ in Panel B becomes higher). The acceleration of investment is consistent with real options intuition since a higher mean reversion speed implies a lower volatility of earnings (see eq. 23 which formalizes this point). Consistently, initial coupons and leverage increase with the speed of mean reversion. Our results thus confirm and elaborate on Gorbenko and Strebulaev (2010) who have also shown that more persistent shocks result in higher leverage ratios.[5] We observe that the relation between leverage and q is positive also at the investment trigger, i.e., when firms enter a stage with no further options available. With respect to leverage dynamics, we observe that an increase in the

[5] We show that the observed higher leverage ratios for high q are driven by different value changes of equity and debt depending on long-term profitability. When long-term profitability is high, a higher q increases both equity and debt, albeit the latter grows faster. When long-term profitability is low, debt increases with q, however, equity decreases.

speed of mean reversion results in a decrease in leverage relative to previous levels when firms exercise their investments with no further follow-on growth options. This effect holds irrespective of long-term profitability level. Credit spreads decrease as the speed of mean reversion increases for both the base case of long-term profitability θ and the case where θ is low, which is also consistent with the fact that a higher q implies lower volatility.

Firm value is increasing in the speed of mean reversion when long-term profitability is high (see base case) and decreasing when long-term profitability is lower. Thus, more generally one should expect a U-shape for intermediate values of long-term profitability. Such a U-shape can be explained with reference to the effect of speed of mean reversion on volatility which shows a similar U-shape relation with leverage.

[Insert Table 1 here]

Firms and investors often consider long-term profitability prospects when making policy or investment decisions.[6] Table 2 shows that an increase in long-term profitability (θ) accelerates investment as indicated by the lower x_I (see panel A) and the higher expected investment costs ($Inv_b(x)$) (see panel B). A higher long-term profitability creates a U-shape with respect to default thresholds v_L and x_b. As expected, higher long-term profitability increases firm value, increases the initial leverage ratio, and reduces credit spreads. The positive relationship between θ and the leverage ratio and the negative relation between θ and credit spreads holds also at the investment threshold. Our results show that when the firm exercises its investment options reaching a state with no further growth options available, leverage decreases with respect to the level long-term profitability. This is driven by the lower threshold where investment takes place when long-term profitability is high which does not allow for high leverage levels at the investment trigger compared to earlier levels.

[Insert Table 2 here]

While long-term profitability has a positive effect on leverage ratios, this is not generally the case with respect to current profitability levels as we show below. Table 3 shows a negative relation

[6] For example there is extensive debate relating the long-term prospects of Tesla: https://www.forbes.com/sites/sergeiklebnikov/2022/06/09/tesla-stock-can-jump-another-50-thanks-to-superior-growth-in-the-years-ahead-analysts-say/?sh=61363ad51f06

between leverage and profitability levels (see panel B) for the base case parameters for a wide range of x values, while for high values of x, the relation becomes positive. Thus, there is an overall U-shape relation between leverage and profitability with x.

[Insert Table 3 here]

Sarkar and Zapatero (2003) have shown in the context of GMR that the relation of leverage and profitability is negative. In contrast, we show that for high current profitability where profitability levels become close to the investment trigger the result may be reversed, and debt values may increase more rapidly compared to equity (hence resulting in an increase in the leverage ratio).[7] We have conducted additional sensitivity analysis showing that the U-shape is robust to alternative parameterizations such as different long-term profitability or different mean-reversion speeds (Online Appendix 4.2. shows additional results). In fact, this relation holds for $q \to 0$ indicating that this U-relation holds also for non-stationary firms (more on this in the next section). Empirical studies usually consider the return on asset instead of the level of profits. Interestingly, one can easily verify that a theoretical constructed measure of return on assets defined as the level of earnings scaled with unlevered assets (Ub(x)) is monotonically increasing in x and so the U-shape holds not only with the level of earnings but also with respect to return on assets with leverage.

What drives this different result in our model compared to Sarkar and Zapatero (2003)? There are two important features of our model that differ compared to Sarkar and Zapatero model. Firstly, we use an AMR instead of a GMR process. In Sarkar and Zapatero (2003) volatility increases with earnings (due to the GMR process) which reduces leverage at high earnings. However, this effect is not present in our context since with an AMR the volatility of earnings remains the same irrespective of the earnings level. Thus, in our model at high profitability levels the relative reduction in risk increases the tax benefits of debt and thus results in higher leverage. This effect is also present in Raymar (1991). However, Raymar (1991) focuses on short-term debt which exasperates the positive effect on leverage of an increase in profitability. Moreover, in Raymar (1991) default is exogenous, while we allow for endogenous default. Secondly, in comparison with Sarkar and Zapatero (2003), our model adds a growth option. We have investigated the relation between leverage and profitability assuming an AMR and no growth option. Online Appendix

[7] Equity value is not reported but can be calculated as the difference between firm value and debt value.

A.4.3. shows that in the absence of a growth option the relation is negative for high x (i.e., the U-shape is not present). Thus, the growth option is the main driver for the U-shape of leverage with profitability.[8]

Panel A also shows that the investment trigger increases as x increases. However, investment is accelerated since the increase in the investment trigger is not as significant compared to the incremental increase in x. Indeed, the acceleration of investment can also be seen by the increase in the expected value of investment costs ($Inv_b(x)$) (see Panel B). In Panel A, we also find that x_b increases with x while v_L is slightly decreasing. These results are driven by firm's choice relating to coupon levels R_0 and R_1 . Indeed, R_0 is increasing whereas R_1 is decreasing in relation with x.

In Panel B, as expected, higher profitability increases firm and debt values and the net benefits of debt. When firms reach the investment trigger and there are no more growth options available the leverage ratio decreases with profitability, albeit slightly. We also observe that the change in the leverage ratio at this stage relative to the initial level follows an inverted U-shape. A higher-level x creates a slight U-shape for credit spreads at t = 0 which is like the one observed for leverage. At the investment trigger credit spreads decrease slightly (following a like the leverage pattern).

We next discuss the impact of the growth option expansion factor and capital investment cost (results not tabulated for brevity). A higher growth expansion factor accelerates investment, delays default, and improves firm value. Despite the increase in the net benefits of debt, the leverage ratio at t = 0 decreases. This result agrees with the well-documented debt conservatism for firms with growth options (see Graham and Harvey, 2001).[9] The negative relation between market leverage and growth options also holds in Hackbarth and Mauer (2012) who used a Geometric Brownian Motion for earnings when the second-best solution (like the one we apply in our model) is used.[10] On the other hand, we find that the leverage ratio increases with the expansion factor when firms

[8] As shown in Sarkar and Zapatero (2004) leverage does not vary with profitability for non-stationary firms when the firm does not have growth options.

[9] It should be noted that coupon levels increase at t = 0 but since the improvement in equity value is more significant than that of debt the leverage ratio decreases. Credit spreads at t = 0 do not show any notable increase which supports a debt "conservatism" argument.

[10] They document a U-shape for the first-best case, i.e., in the absence of agency conflicts. We do not analyze the first-best case in this paper since it reflects the hypothetical scenario with no conflicts of interest between debt and equity holders. This analysis would have been needed to measure agency costs, an issue we do not investigate in this paper. Ogden and Wu (2013) show empirical evidence that the relation between leverage and growth options (market-to-book) is convex.

reach a stage where they exercise their investment growth options. There is also a more notable increase in credit spreads at this latter stage. Opposite directional effects to the one discussed above for the expansion factor are observed with respect to capital investment cost level (*I*).

In Appendix A.4.1. we present sensitivity results with respect to the optimal priority rule. We find like in Hackbarth and Mauer (2012) that under a "me-first" priority rule for initial debt firm values are improved and that there is higher debt conservatism and underinvestment.

3.2. Empirical predictions and hypotheses

Table 4 summarizes the model predictions regarding firms with mean reversion in earnings aiming to provide guidance for empirical work in the area. In the next section we further facilitate this step by providing the framework for estimating the parameters of the continuous time process and provide empirical cross-sectional evidence for both firms with mean reverting and non-stationary earnings.

[Insert Table 4 here]

Initial financing (Lev_b) predictions reflect what one would expect to see in the cross section within a setting involving future growth option potential. For example, it is reasonable to expect a negative relation between market to book and leverage (as predicted at Lev_b) in the cross section than a positive one (as in Lev_T) which would only exist if growth options following an investment decision completely disappear.[11]

We next summarize our main empirical hypothesis that are based on the predictions of our model with respect to leverage for mean reverting firms (see Lev_b in Table 4).

H1: For mean reverting firms, the relation between market leverage during the timing of investment using debt financing in the presence of future growth options is negative with respect to volatility, positive with respect to mean reversion speed and long-term profitability, U-shaped with profitability and decreasing with the value of growth opportunities.

[11]An addition of one more stage T+1 would imply that that firms' decisions relating Lev_T would now consider the effect of future growth options and hence by backward induction a higher growth in a last stage would lead to a reduction in leverage at T. Hence we expect Lev_b to be representative of the prediction of the relation of leverage with other factors in the presence of future growth options.

Within a similar context, Hackbarth and Mauer (2012) developed a model for firms having non-stationary earnings (GBM motion). Within our setting the non-stationary case reflects the case where the speed of mean reversion q tends to zero. We find that for non-stationary firms, the relation between market leverage during the timing of investment using debt financing in the presence of future growth options is negative with respect to volatility, U-shaped with profitability and decreasing with respect to the value of growth opportunities. These predictions are verified in sensitivity results we have conducted of the Hackbarth and Mauer (2012) setting. Indeed, our sensitivity analysis in Online Appendix section A.4.2 shows that a U-shape profitability-leverage relation holds also for firms following GBM in earnings firms. We have also conducted sensitivity approximating the non-stationary case within our context which can be obtained when q tends to zero verifying a similar U-shape exists within our setting for very low q.[12] In sum, our analysis above clarifies that the predictions with respect to firms with non-stationary earnings are similar with those of firms with mean reversion. However, within the mean reversion sample we can also test the relation with estimated parameters relating long-term profitability and mean reversion speed.

4. Empirical analysis

4.1. Mean reversion process estimation and descriptive statistics for mean-reverting and non-stationary firms

In this section we show how to estimate the parameters of the continuous time process. We also investigate the prevalence of earnings mean reversion and non-stationarity in the data and provide descriptive statistics for the two subsamples.

The continuous time model dynamics for the earnings in equation (1) need to be translated in a suitable discrete time approximation for estimation. To do that we first note that the solution of the stochastic differential equation (SDE) in equation (1) is of the following form (see Lo and Wang, 1995):

[12] Formally our case $q \to 0$ which define non-stationary firm does not nest the GBM motion. However, our qualitative (sign) empirical predictions of our case $q \to 0$ are the same as one would obtain with assumption a GBM motion process on earnings. It should also be noted that throughout our analysis we use the assumption of second-best investment and financing of the growth option as discussed also in Hackbarth and Mauer (2012) because this is likely the case that holds in practice.

$$x_t = x_0 e^{-qt} + \theta(1 - e^{-qt}) + \sigma \int_0^t e^{q(s-t)} dz_s \tag{21}$$

The conditional expected value and variance of x_t can be obtained by solving the Kolmogorov forward equation (as in Dixit and Pindyck, 1994, p.90). These moments have also been derived in Lo and Wang, (1995). The conditional expected value of earnings is:

$$E(x_t) = x_0 e^{-qt} + \theta(1 - e^{-q}\ ) = \theta + (x_0 - \theta)e^{-qt} \tag{22}$$

Note that for $q > 0$ as $t \to \infty$, $E(x_t) \to \theta$ which confirms the mean-reverting nature of the process. The variance of the variable *x* is also obtained as follows:

$$Var(x_t) = \frac{\sigma^2}{2q}(1 - e^{-2qt}) \tag{23}$$

Note that as *t* becomes large an increase in mean reversion speed q decreases the variance of *x*.

The solution of the SDE implies the following discrete version of solution can be used to generate the dynamics of *x* (see Dixit and Pindyck, 1994, p.76, eq.19):

$$\Delta x_t = \theta(1 - e^{-q}) + (e^{-q} - 1)x_{t-1} + \sigma\sqrt{\frac{1-e^{-2q}}{2q}} Z_t \tag{24}$$

where $Z_t \sim N(0,1)$. The above specification implies that the error volatility (we call "temporal variation") per unit of interval is:

$$\sigma_\varepsilon = \sigma\sqrt{\frac{1-e^{-2q}}{2q}} \tag{25}$$

Note that equation (24) can also be used to simulate the stochastic dynamics of the continuous process. To estimate the mean reversion speed (q), long-term mean (θ) and volatility (σ) in equation (24) we estimate the following AR (1) model in discrete time (see Dixit and Pindyck, 1994, p.76):

$$\Delta x_t = a + bx_{t-1} + \varepsilon_t \tag{26}$$

We then associate the estimated constant, slope and error term volatility of eq. (26) with the continuous time model approximation analogue in equation (24) which results in the following solution for the parameters:

$$q = -\ln(1 + \hat{b}) \tag{27}$$

$$\theta = -\frac{\hat{a}}{\hat{b}} \quad (28)$$

$$\sigma = \sigma_{\varepsilon}\sqrt{\frac{-2\ln(1+\hat{b})}{(1+\hat{b})^2 - 1}} \quad (29)$$

For the dynamics above to be meaningful we need that $-1 < \hat{b} < 0$ so that we obtain a positive mean reversion speed. Note that the smaller the coefficient $\hat{b}$ the larger the speed of mean reversion q while as $\hat{b} \to 0$ we have $q \to 0$. To avoid $\hat{b} = 0$ and ensure that earnings dynamics in eq. (26) remains stationary we employ an Augmented Dickey-Fuller. We estimate eq. (26) after first de-seasonalizing the earnings series and then used an Augmented Dickey-Fuller test to test the null of non-stationary series $\hat{b}$=0 versus the alternative of stationary series $\hat{b}$<0 using a 5% level of significance. Note that σ_{ε} can be estimated using the standard error of regression of eq. (26). We use this as an input to estimate our first risk measure in eq. (29) which we subsequently refer as "risk1". σ_{ε} can be used as an alternative measure of risk of temporal variation (we subsequently refer to this as "risk2"). We also use σ_{ε} as a measure of risk for the firms found to be non-stationary. Volatility measures are then scaled with total assets since these estimates refer to the level of the earnings process.

Our initial sample is from the quarterly COMPUSTAT database between 1961 and 2019. We exclude financial firms (Standard Industrial Classification (SIC) codes 6000 to 6999) and regulated firms (SIC codes 4900 to 4999). We require that a firm is included in the analysis for testing for mean reversion if it has at least 40 consecutive quarterly observations (10 year of data) for earnings (oibdpq).

The total number of non-financial and non-regulated firms in the sample before and after the requirement of at least 40 consecutive observations is shown in Table 5. The table also shows the number of firms classified as mean reverting using the full sample available for each firm by applying the Augmented Dickey-Fuller to test the null of non-stationary series (based on estimating eq. 26).

[Insert Table 5 here]

Figure 1 illustratively shows a selection of two firms from our sample, one found to be mean-reverting and one which is non-stationary. Clearly, firm 1 earnings revert to a long-term mean whereas firm 2 appears not to revert to a long-term mean level.

[Insert Figure 1 here]

Using the Augmented Dickey-Fuller our analysis shows that 60% of the firms with available data are classified as mean reverting (stationary). Since very little is known about the long-term profitability, the mean reversion speed, and the volatility of earnings for firms following a mean reverting process, as well as their characteristics we first provide some descriptive statistics.

Our sample is constructed by merging the COMPUSTAT financial data with CRSP. First, for our sample of mean reverting firms we create rolling estimates of long-term profitability, mean reversion speed and volatility for each additional available firm quarter observation setting forty consecutive observations as the minimum.[13] We then merge the dataset based on the specific year-quarter with CRSP monthly security prices based on the initial month of the same quarter. We calculate several variables including market leverage (gross, net and book), profitability (ROA), volatility scaled by assets, size (log sales), market to book, tangible assets, cash ratio, capital expenditure to assets and industry concentration based on Herfindahl index (see Danis et a., 2014 and Eckbo and Kisser, 2021). After trimming the 1% lowest and highest values for variables to avoid outliers we are left with 1,683 unique mean reverting firms with complete data and a total of 41,067 firm-quarter year observations on these variables. We apply the same process for the sample of non-stationary firms, albeit we only now create rollover estimates of volatility based on the standard error of the regression in eq. (26). This leaves us with a sample of 1,397 unique non-stationary firms with complete data and a total of 38,368 firm-quarter year observations on all variables.[14]

Table 6a provides information on the prevalence of earnings mean reversion in different industries.

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[13] This induces some autocorrelation in the estimates. We have controlled for possible autocorrelation in the multivariate analysis in the standard error of estimates. We have also unreported results with a rolling window of five observations which provide similar insights as the one reported.

[14] We filter the 1% lowest and highest values for gross market leverage (total), net leverage total, book leverage, cash ratio, capital to assets (year-to-date), mean reversion speed, size, profitability, logarithm of volatility based on estimate of eq. (29) (for mean-reverting) or logarithm of the standard error from regression in eq. (26) (for non-stationary), market-to-book, tangible assets, and long-term profitability. This process also ensures that leverage ratios and other ratios such as tangible assets remain within the unit interval.

[Insert Table 6a here]

The data show that most mean reversion observations, with about 75% of our sample, belong to the manufacturing sector, followed by services with about 19% and retail trade with about 17%. The mining sector (a natural candidate for mean reversion) only comes fourth with about 12%. We find that mean reversion also exists in other sectors such as transportation, communications, electric, gas and sanitary service and wholesale trade (with about 11% and 6% respectively).

Table 6b shows a similar breakdown for our sample of non-stationary firms. Similarly, most firm-year quarter observations (albeit less compared to the mean reverting sample) of about 51% are in the manufacturing sector. Similarly with mean reverting sample this is followed by services with about 19%. About 15% of our sample of non-mean reverting are within the retail, mining, and wholesale trade sector (with about 5%-6% in each of these sectors respectively).

Table 7a provides some further descriptive statistics for our sample of mean reverting firms while Table 7b provides descriptive statistics for non-stationary firms.

[Insert Table 7a here]

[Insert Table 7b here]

Our sample of mean reverting firms has some different characteristics compared to a broader sample that includes all firms (both mean reverting and non-stationary). Although our sample covers a wider period compared to Danis et al. (2014) (see Table 1 of Danis et al., 2014, p.431), our median and mean reverting firm are more leveraged, of significantly smaller size based on log(sales) and with less growth potential based on market to book. However, mean reverting firms in our sample appear to have more tangible assets. This may reflect the fact that there is a large concentration of mean reverting firms in the manufacturing sector. We also find that our sample of mean reverting firms have more industry concentration compared to the overall sample of firms as shown in Danis et al. (2013). Finally, we note that the profitability exhibits a lower standard deviation for our sample of mean reverting firms. The descriptive statistics of our sample of non-stationary firms (see Table 9b) confirms that the above differences between a sample of mean-reverting and non-stationary firms hold also within our subsamples. This provides assurance that despite the restrictions that we have imposed for estimation of earnings process parameters, the samples remain representative of the overall populations for each category.

For our sample of mean reverting firms, we estimate two risk measures, one based on equation (29) (which uses regression eq. (26) standard error as input), and one based on temporal variation which is based on the standard error of model described in equation (26) scaled by assets. Both produce similar results with median volatility scaled by assets estimates of about 1.1%. In fact, we have also found that these two risk measures have a correlation of 97%. Our final two estimated variables in Table 7a present information about the estimates of long-term profitability and mean reversion speed. We find that mean and median long-term profitability to assets is close but slightly lower than the corresponding mean and median profitability (is about 2% of assets). The median mean reversion speed is high showing that most firms revert quickly to their long-term means. For non-stationary firms the only relevant stochastic process related variable is the one based on the standard error estimate of equation (26) (see Table 7b). We find that the risk based on this estimate has similar mean and median when scaled by total assets as the corresponding estimate for our sample of mean reverting firms.

Table 8 presents median empirical estimates of our theoretical model parameters which have been defined relative to a normalized earnings level. We observe that empirically the median firm has a long-term profitability which is about 75% of earnings which explains why we have also focused our theoretical model sensitivity results on this case.

[Insert Table 8 here]

The median estimated volatility relative to the earnings level is about 39% which is very close to our base case used in the theoretical analysis where volatility relative to normalized earnings was at 40%. Finally, the median mean empirical estimate of reversion speed is higher compared to what we used in our base case in our theoretical model sensitivity results. However, in our theoretical analysis we allowed for lower mean reversion speeds to investigate cases where growth options are not immediately exercised and thus we are able to provide insights relating to a combination of high/low levels of mean reversion speed with high/low long-term profitability (see Table 2).

4.2. Multivariate analysis

In this section we empirically investigate the relation between different factors affecting leverage for our sample of mean reversion firms and non-stationary firms. For the multivariate empirical tests, we run the following linear regression for mean reverting firms:

$$L_{i,t} = a + \gamma_0 \Pi_{i,t-1} + \gamma_2 Risk_{\iota,t-1} + \gamma_3 MB_{\iota,t-1} + \gamma_1 \theta_{i,t-1} + \gamma_2 q_{i,t-1} + \beta X_{i,t-1} \quad (30)$$

where $L_{i,t}$ denotes gross market leverage (using total debt), $\Pi_{i,t-1}$ is operating income after depreciation over total assets, $Risk_{\iota,t-1}$ is the estimated measure of earnings risk (see eq. 29) scaled by total assets, $MB_{\iota,t-1}$ is market-to-book, $\theta_{i,t-1}$ denotes the estimated long-term profitability (see eq. 28) scaled by total assets and $q_{i,t-1}$ the estimated mean reversion speed (see eq. 27). $X_{i,t-1}$ is a set of controls which includes size, tangible assets and Herfindahl industry concentration.

For non-stationary firms we run a similar specification as in equation (30), albeit $Risk_{\iota,t-1}$ is the estimated measure of earnings risk arising from the standard error of the regression eq. (26) scaled by total assets. In addition, we exclude $\theta_{i,t-1}$ and $q_{i,t-1}$ which is not applicable for non-stationary firms.

Note that in contrast to Danis et. al. (2014) and Eckbo and Kisser (2021) we do not investigate refinancing events in this specification. The reason is twofold. First, our first goal is to see how our newly introduced variables based on the estimates of the stochastic process perform unconditionally to investment events. We also investigate how traditionally control variables (e.g., market-to-book and size) behave for our sub-samples of mean reverting and non-stationary firms. Secondly, and more importantly, Danis et. al. (2014) and Eckbo and Kisser (2021) focus on testing refinancing events. Our theoretical framework, on the other hand, focuses on adjustments in leverage during lumpy investment decisions. To confront our theoretical model with the data we will thus need to focus on investment related debt rebalancing events (we investigate this subsequently in the paper).

Table 9 provides the results of empirically investigating the relation between factors affecting leverage by estimating eq. (30) above.

[Insert Table 9 here]

In model (1) which focuses on mean-reverting firms, the results show that leverage is negatively associated with profitability, risk, and market-to-book (a proxy for growth options) and positively related to long-term profitability and mean reversion speed. Our results also show similarly to a sample which includes both mean reverting and non-stationary firms (see e.g., Danis et al. 2014, Table 2, p.433) that leverage is positively related to size and tangible assets. In contrast to Danis et al., (2014) we find Herfindahl industry concentration to negatively affect leverage.[15]

Model (2) which focuses on the sample of non-stationary firms also shows that leverage is negatively associated with profitability. Interestingly, the negative effect appears even more pronounced. Leverage is also negatively associated with risk and market-to-book as expected. In addition, the size and tangible control variables appear with the expected sign. Herfindahl shows to be insignificantly associated with leverage.

The previous analysis is unconditional on investment events with debt financing and thus is hard to identify which of the effects are caused by dynamic adjustments in leverage or simply due to firms' inaction in frequently adjusting leverage. Thus, in contrast to Tserlukevich (2008) who focuses on inaction due to irreversible costs and investment options, our theoretical model predicts the relation of leverage and profitability *at* investment events involving debt financing should be U-shaped.

In Table 10 we estimate model (30) now focusing only on investment events involving debt financing defined as firm-quarter periods where a firm's year-to-date investment to assets ratio exceeds the 75% quartile year-to-date industry ratio *and* the long-term book debt ratio exceeds the 75% quartile industry ratio in a given period.[16] This leaves us with 3,225 investment events with debt financing for a sample of 518 firms for the mean reverting sample and 3,004 investment events with debt financing for 409 firms that are non-stationary. In order to investigate the U-shape relation of profitability with leverage the analysis now includes the square terms of profitability, as well as an interaction term of square profitability with market-to-book.

[15] We note that our industry concentration measure uses a two-digit SIC code whereas Danis et al. (2014) use a four-digit SIC.

[16] We have considered alternative characterizations like for example focusing on comparison with median industry measures or comparing firms' own median or 75% quartile investments and debt financing levels with similar results. Intuitively, our focus on high long-term ratios is in an attempt to better capture rebalancing of financing related to investment related financing.

[Insert Table 10 here]

Two key findings from Table (10) are noted. First, we observe that there is a negative relation of profitability with leverage also during investment events. We find that this negative relation becomes more pronounced for high profitability levels as shown by the negative coefficient of the square term of profitability. Thus, we are unable to confirm a U-shape of profitability with leverage. However, our model predicts that the U-shape is driven by growth options. Indeed, we observe a positive sign of the interaction term of profitability with market-to-book which shows that the negative relation of profitability with leverage at high profitability levels is mitigated for firms with more valuable growth options. Purnanandam and Rajan (2018) analysis suggests that firms exercising their growth options translate growth options to assets in place which reduces leverage by also reducing the information asymmetry of equity. However, their study documents the negative relation of leverage with unexpected changes in capital expenditure not the leverage-profitability relationship as studied here.

Our results also show a consistent negative relation of market to book with leverage at investment points for both mean reverting and non-stationary firms. In addition for the sample of mean reverting firms the relation of long-term profitability and mean reversion speed with leverage is positive also at investment events (in line with H1). In terms of control variables, we find that tangible assets' relation with leverage remains positive (as expected) and Herfindahl has negative sign as in the unconditional regression results with the effect however now being stronger for non-stationary firms.

Two effects do not align well with the theoretical trade-off model predictions. First, we notice that risk relations with leverage are not as expected. For non-stationary firms risk does not pick any statistical significance. On the other hand, the positive sign of risk on leverage at investment events for non-stationary firms is not expected by theoretical dynamic capital structure models (see also Hackbarth and Mauer (2012) and the summarized predictions in section 3.2.1.).

Secondly, although the size variable is not directly related with our model predictions, we note the negative effect of size at investment events involving debt financing. Kurshev and Strebulaev (2015) attempt to disentangle the effect of size on leverage on theoretical grounds showing that size may be negatively related with leverage at rebalancing events due to less frequent and more substantial debt adjustments of small firms. In our case, where size is measured as logarithm of sales, size may be capturing effects related to profitability. Indeed, firms wait until higher levels of profitability to trigger investment which should coincide with larger size (when size is measured with sales as here).

Conclusion

We develop a dynamic trade-off model with mean-reversion in earnings and dynamic leverage adjustments at investment points. We provide novel insights on the impact of earnings dynamics and in particular long-term profitability, mean reversion speed and volatility on firm value, leverage levels and dynamic changes in leverage and credit spreads. Our framework also provides various managerial implications regarding the optimal timing of investment and default decisions related to model parameters and the characteristics of the earnings process.

Our empirical investigation shows that a large sample of US firms are characterized as mean reverting. We have provided descriptive statistics for both the sample of mean reverting firms and non-stationary firms and provide estimates of long-term profitability, mean reversion speed and volatilities for mean reverting firms and volatilities for non-stationary firms. Our multivariate analysis confirms a negative relation between our estimated volatility measure with leverage and a positive relation of leverage with long-term profitability and mean reversion speed. However, the relation of leverage with profitability is found negative both unconditionally as well as conditional on investment events for both mean reverting and non-stationary firms. Firms with growth options exhibit higher leverage at high profitability, however the negative relation of profitability with leverage is not fully reversed. Future work could further examine the dynamic capital structure decisions of mean reverting versus non-stationary firms in an attempt to further address the discrepancies between theoretical models and practice as discussed in Graham (2022). For example, one could investigate issues relating to how managerial conservatism affects firms' leverage decisions.

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Table 1. Sensitivity analysis with respect to mean reversion speed (q)

Panel A: Optimal coupon and thresholds

Base case (θ = 1)

q	R_0	R_1	x_I	v_L	x_b
0.10	**0.52**	**0.55**	**1.341**	**-1.501**	**-1.191**
0.15	0.65	0.45	1.238	-1.795	-1.209
0.20	0.78	0.45	1.149	-1.810	-1.134
0.25	0.88	0.47	1.077	-1.738	-1.053
0.30	0.96	0.49	1.021	-1.623	-0.958

Case with lower long-term profitability (θ = 0.75)

q	R_0	R_1	x_I	v_L	x_b
0.10	**0.36**	**0.54**	**1.473**	**-1.356**	**-1.094**
0.15	0.39	0.4	1.409	-1.772	-1.186
0.20	0.46	0.37	1.342	-1.947	-1.169
0.25	0.52	0.39	1.276	-1.969	-1.137
0.30	0.57	0.42	1.217	-1.910	-1.094

Panel B: Values at t = 0

Base case (θ = 1)

	Values at t = 0							Values at investment trigger T		
q	Fb(x)	Db(x)	Ub(x)	NBb(x)	Lev_b	Inv_b	Cr_b	Lev_T	ΔLev	Cr_T
0.10	**20.556**	**7.997**	**26.720**	**1.692**	**0.389**	**7.856**	**0.0050**	**0.485**	**0.096**	**0.0052**
0.15	20.771	10.486	27.234	2.174	0.505	8.637	0.0020	0.545	0.040	0.0020
0.20	21.166	12.787	27.683	2.707	0.604	9.224	0.0010	0.628	0.024	0.0010
0.25	21.524	14.526	28.019	3.139	0.675	9.634	0.0006	0.698	0.023	0.0006
0.30	21.814	15.898	28.253	3.469	0.729	9.908	0.0004	0.752	0.023	0.0004

Case with lower long-term profitability (θ = 0.75)

	Values at t = 0							Values at investment trigger T		
q	Fb(x)	Db(x)	Ub(x)	NBb(x)	Lev_b	Inv_b	Cr_b	Lev_T	ΔLev	Cr_T
0.1	**16.063**	**5.239**	**21.488**	**1.167**	**0.326**	**6.591**	**0.0087**	**0.431**	**0.104**	**0.0089**
0.15	15.227	6.118	21.037	1.261	0.402	7.072	0.0037	0.440	0.038	0.0041
0.2	14.954	7.448	20.950	1.573	0.498	7.568	0.0018	0.499	0.001	0.0019
0.25	14.876	8.530	21.052	1.897	0.573	8.073	0.0010	0.569	-0.004	0.0010
0.3	14.863	9.404	21.207	2.176	0.633	8.521	0.0006	0.633	0.001	0.0006

Notes: Initial earnings level x = 1, risk-free rate r = 0.06, tax rate τ = 0.15, proportional bankruptcy costs b = 0.5, growth option rate e = 2, investment cost I = 10. For the mean-reverting stochastic model parameters we use σ = 0.4, long-term mean θ = 1 or θ = 0.75 and vary mean reversion speed q. ΔLev stands for change in leverage and is calculated as Lev_T - Lev_b. Base case parameters highlighted in bold.

Table 2. Sensitivity analysis with respect to long-term profitability (θ)

Panel A: Optimal coupon and thresholds

θ	R_0	R_1	x_I	v_L	x_b
0.6	0.31	0.53	1.571	-1.217	-0.974
0.75	0.36	0.54	1.473	-1.356	-1.094
1	**0.52**	**0.55**	**1.341**	**-1.501**	**-1.191**
1.25	0.73	0.58	1.237	-1.544	-1.234
1.5	0.99	0.61	1.156	-1.504	-1.210
1.75	1.29	0.63	1.093	-1.411	-1.132
2	1.64	0.62	1.047	-1.282	-0.976
2.25	2.03	0.57	1.017	-1.161	-0.764

Panel B: Values at t = 0

	Values at t = 0							Values at investment trigger T		
θ	Fb(x)	Db(x)	Ub(x)	NBb(x)	Lev_b	Inv_b	Cr_b	Lev_T	ΔLev	Cr_T
0.60	13.635	4.246	18.243	0.970	0.311	5.578	0.0130	0.405	0.094	0.0126
0.75	16.063	5.238	21.488	1.167	0.326	6.591	0.0087	0.430	0.104	0.0090
1	**20.556**	**7.997**	**26.720**	**1.692**	**0.389**	**7.856**	**0.0050**	**0.485**	**0.096**	**0.0052**
1.25	25.429	11.561	31.741	2.396	0.455	8.708	0.0031	0.548	0.093	0.0032
1.5	30.532	15.923	36.579	3.214	0.522	9.261	0.0022	0.611	0.089	0.0022
1.75	35.763	20.952	41.283	4.093	0.586	9.613	0.0016	0.667	0.081	0.0016
2	41.063	26.787	45.883	5.004	0.652	9.824	0.0012	0.716	0.064	0.0012
2.25	46.398	33.325	50.414	5.928	0.718	9.944	0.0009	0.754	0.036	0.0009

Notes: Initial earnings level x = 1, risk-free rate r = 0.06, tax rate τ = 0.15, proportional bankruptcy costs b = 0.5, growth option rate e = 2, investment cost I = 10. For the mean-reverting stochastic model parameters we use σ = 0.4, mean reversion speed q = 0.1 and vary long-term mean of earnings θ. ΔLev stands for change in leverage and is calculated as Lev_T - Lev_b. Base case parameters highlighted in bold.

Table 3. Sensitivity analysis with respect to current profitability level (x)

Panel A: Optimal coupon and thresholds

x	R_0	R_1	x_I	v_L	x_b
-0.40	0.30	0.8	1.323	-1.481	-1.660
-0.25	0.32	0.8	1.324	-1.481	-1.616
0	0.37	0.7	1.327	-1.481	-1.508
0.25	0.41	0.7	1.330	-1.481	-1.422
0.50	0.44	0.6	1.332	-1.481	-1.358
0.75	0.48	0.6	1.336	-1.501	-1.274
1	**0.52**	**0.6**	**1.341**	**-1.501**	**-1.191**
1.25	0.59	0.5	1.352	-1.501	-1.049
1.30	0.62	0.45	1.357	-1.501	-0.989
1.35	0.68	0.4	1.369	-1.481	-0.873

Panel B: Values at t = 0

	Values at t = 0							Values at investment trigger T		
x	Fb(x)	Db(x)	Ub(x)	NBb(x)	Lev_b	Inv_b	Cr_b	Lev_T	ΔLev	Cr_T
-0.40	10.391	4.583	13.440	1.176	0.441	4.225	0.0055	0.492	0.051	0.0054
-0.25	11.372	4.912	14.596	1.225	0.432	4.449	0.0051	0.491	0.059	0.0054
0	13.039	5.688	16.576	1.318	0.436	4.855	0.0051	0.491	0.055	0.0053
0.25	14.762	6.310	18.716	1.407	0.427	5.361	0.0050	0.491	0.064	0.0054
0.50	16.563	6.780	21.069	1.493	0.409	6.000	0.0049	0.490	0.081	0.0053
0.75	18.476	7.393	23.688	1.589	0.400	6.801	0.0049	0.486	0.086	0.0052
1	**20.556**	**7.997**	**26.720**	**1.692**	**0.389**	**7.856**	**0.0050**	**0.485**	**0.096**	**0.0052**
1.25	22.882	9.050	30.322	1.807	0.396	9.247	0.0052	0.484	0.088	0.0052
1.30	23.389	9.509	31.118	1.832	0.407	9.561	0.0052	0.483	0.077	0.0052
1.35	23.915	10.418	31.908	1.856	0.436	9.848	0.0053	0.485	0.050	0.0052

Notes: Risk-free rate r = 0.06, tax rate τ = 0.15, proportional bankruptcy costs *b* = 0.5, growth option rate *e* = 2, investment cost *I* = 10. For the mean-reverting stochastic model parameters we use σ = 0.4, mean reversion speed q = 0.1 and long-term mean of earnings θ = 1. ΔLev stands for change in leverage and is calculated as Lev_T - Lev_b. Base case parameters highlighted in bold.

Table 4. Summary of directional effects on firm value, firm investment and default policies and leverage dynamics

Parameter	Fb(x)	x_I	v_L	x_b	Lev_b	Lev_T	ΔLev
Volatility (σ) [1]	U	+	-	-	-	-	+
Speed of mean reversion (q)[2]	U	-	U	U	+	+	-
Long-term profitability (θ)	+	-	U	U	+	+	-
Current earnings level (*x*)	+	+	-	+	U	Moderate -	Inverted U
Growth expansion factor (e)	+	-	-	-	-	+	+
Capital investment cost (I)	-	+	+	+	+	-	-
Me-first priority rule (compared with equal priority)	+	-	-	-	-	Minor positive/No significant change	+

Notes: The above table summarizes our sensitivity results. Note 1: the relation holds for "average" long-term profitability: firm value is increasing when long-term profitability is low and decreasing when long-term profitability is high. All other effects of volatility remain unchanged. Note 2: When long-term profitability is low (high) firm value is decreasing (increasing).

Table 5. Sample of mean reverting and non-stationary firms.

All COMPUSTAT firms including financial and regulated	38,205
Financial firms	11,053
Regulated	929
Total non-financial and non-regulated firms	26,223
Number of firms with N = 40 consecutive earnings (oibdpq)	5,321
N. of firms classified as mean reverting	3,197
N. of unique mean reverting firms with complete observations on all variables (with 1% highest and lowest outliers removed)	1,683
Total number of firm-quarter year observation with full data for mean reverting firms	41,067
N. of unique non-mean reverting firms with complete observations on all variables (with 1% highest and lowest outliers removed)	1,397
Total number of firm-quarter year observation with full data for non-mean reverting firms	38,368

Table 6a. Industry classification of mean reverting firms

SIC	Division	Firms-Yeas obs.	Perc.
0100-0999	Agriculture, Forestry and Fishing	286	1.00%
1000-1499	Mining	3532	12.38%
1500-1799	Construction	618	2.17%
2000-3999	Manufacturing	21392	74.97%
4000-4999	Transportation, Communications, Electric, Gas and Sanitary service	3122	10.94%
5000-5199	Wholesale Trade	1811	6.35%
5200-5999	Retail Trade	4829	16.92%
7000-8999	Services	5393	18.90%
9900-9999	Nonclassifiable	84	0.29%
	Total	41,067	100%

Table 6b. Industry classification of non-mean reverting firms

Division	Firms-Yeas obs	Perc.
Agriculture, Forestry and Fishing	64	0.17%
Mining	2201	5.74%
Construction	541	1.41%
Manufacturing	19540	50.93%
Transportation, Communications, Electric, Gas and Sanitary service	4687	12.22%
Wholesale Trade	1835	4.78%
Retail Trade	2072	5.40%
Services	7340	19.13%
Nonclassifiable	88	0.23%
Total	38,368	100%

Notes: The table provides the industry (SIC) classification of all non-financial, non-regulated firms with at least 40 consecutive earnings (oibdpq) which are classified as either mean reverting or non-mean reverting (non-stationary) and have complete data on a set of variables needed for the analysis (see Table 5).

Table 7a. Descriptive statistics for sample of mean reverting firms

	Mean	Median	Std
Market Leverage (gross)- Total	0.287	0.235	0.224
Market Leverage (gross)- LT debt	0.247	0.191	0.220
Market Leverage (Net) -Total	0.178	0.170	0.334
Book leverage -Total	0.262	0.237	0.181
Book leverage- LT debt	0.216	0.189	0.175
Profitability (ROA)	0.025	0.027	0.030
Risk1 (Sigma to assets)	0.017	0.011	0.024
Risk2(Temporal variation to asset)	0.017	0.011	0.020
Size (Log of sales)	-1.180	-1.172	2.005
Market to book	1.152	0.930	0.800
Tangible assets	0.336	0.276	0.242
Herfindahl index	0.293	0.238	0.197
Cash ratio	0.107	0.062	0.124
Capex to assets (year-to-date)	0.034	0.021	0.038
Capex to assets (quarterly)	0.000	0.006	0.040
LT profitability to assets	0.020	0.021	0.019
Mean reversion speed (q)	0.726	0.517	0.637

Table 7b. Descriptive statistics for sample of non-mean reverting firms

	Mean	Median	Std
Market Leverage (gross)- Total	0.240	0.182	0.208
Market Leverage (gross)- LT debt	0.221	0.162	0.205
Market Leverage (Net) -Total	0.163	0.128	0.261
Book leverage -Total	0.276	0.256	0.184
Book leverage- LT debt	0.244	0.223	0.180
Profitability (ROA)	0.037	0.036	0.024
Risk2(Temporal variation to asset)	0.007	0.005	0.009
Size (Log of sales)	-0.304	-0.266	1.659
Market to book	1.599	1.287	1.095
Tangible assets	0.314	0.237	0.243
Herfindahl index	0.281	0.203	0.215
Cash ratio	0.113	0.066	0.130
Capex to assets (year to date)	0.034	0.021	0.036
Capex to assets (quarterly)	0.000	0.006	0.036

Notes: The table provides descriptive statistics of all non-financial, non-regulated firms with at least 40 consecutive earnings (oibdpq) which are either mean reverting or non-stationary and have complete data on a set of variables needed for the analysis (see Table 7). Gross market leverage defined as long-term debt plus short-term debt divided long-term debt plus short-term debt plus the market value of equity, with market value of equity calculated using CRSP data (dlttq+dlcq)/ (dlttq+dlcq+price x shrout) while gross LT leverage excludes short-term debt. Net leverage total is defined as book debt net of cash holdings (dlttq+dlcq -cheq) divided by the sum of book debt net of cash and market equity. Profitability is defined as operating income after depreciation over total asset (oibdpq/atq). Book leverage is defined as book debt scaled by total assets while book LT debt excludes short-term debt. Size is the inflation adjusted log of sales (log(salesq/cpiind)). Risk1 is defined as the rollover estimate of volatility as defined in eq. (29) scaled by total assets of the year-quarter. Risk2 is defined as the rollover estimate of volatility of the standard error of eq. (26) scaled by total assets of the year-quarter. Market-to-book is the market value of equity plus long-term debt divided by total assets ((equity+dlttq)/atq). Tangible assets are property plant and equipment over total assets (ppentq/atq). Herfindahl index a particular industry is defined as the sum of squared market shares for all firms in a two-digit SIC industry in a year-quarter. The market share of a firm is defined as sales of that firm in a year-quarter divided by the total sales in the industry of firm in that year-quarter. Cash ratio is cash and equivalents (cheq) scaled by total assets. Capex to assets is (capex) scaled by total assets (year-to-date or quarterly). LT profit to asset is rollover estimated long-term profit of a firm in a year-quarter as defined in eq. (28) divided by total assets of the corresponding year-quarter. Mean reversion speed is the rollover estimated mean reversion speed as defined in eq. (27) in a year-quarter scaled by total assets.

Table 8. Empirical estimates of theoretical model parameters of mean reverting process

	Median
LT profitability to earnings (θ/x)	0.75
Volatility (sigma) to earnings (σ/x)	0.39
Mean reversion speed (q)	0.52

Notes: The table provides the implied theoretical model parameters arising from empirical median sample estimates. Long -term profitability to earnings (θ/x) is the median sample estimate of long-term profitability to assets (see Table 9a) divided by the median sample estimate of profitability (from Table 7a). Similarly, volatility (sigma) to earnings (σ/x) is the median sample estimate of long-term sigma to assets (see Table 7a) divided by the median sample estimate of profitability (from Table 7a). Mean reversion speed (q) is the median estimate of mean reversion speed obtained from Table 9a.

Table 9. Factors affecting leverage for mean reverting firms

	Mean reverting		Non-stationary	
	(1)		(2)	
Profitability	-1.115	***	-1.431	***
	(0.049)		(0.057)	
Risk	-0.583	***	-0.733	***
	(0.077)		(0.125)	
Market-to-book	-0.107	***	-0.076	***
	(0.002)		(0.001)	
LT profit to assets	0.155	*		
	(0.086)			
Mean reversion speed	0.037	***		
	(0.002)			
Controls:				
Size	0.011	***	0.008	***
	(0.001)		(0.001)	
Tangible	0.149	***	0.153	***
	(0.009)		(0.01)	
Herfindahl	-0.051	***	-0.018	
	(0.01)		(0.012)	
Quarter FE	Yes		Yes	
Industry FE	Yes		Yes	
R^2	0.345		0.43	
Adj. R^2	0.341		0.427	
Firm obs.	1,683		1,397	
Total obs.	41,067		38,368	
F-test (Prob)	<0.0001		<0.0001	

Notes: The analysis is based on all non-financial, non-regulated firms with at least 40 consecutive earnings (oibdpq) which are mean reverting (see Table 7). The dependent variable is gross market leverage defined as long-term debt plus short-term debt divided long-term debt plus short-term debt plus the market value of equity, with market value of equity calculated using CRSP data (dlttq+dlcq)/ (dlttq+dlcq+price x shrout). Profitability is defined as operating income after depreciation over total asset (oibdpq/atq). Size is the inflation adjusted log of sales (log(salesq/cpiind)). Risk is defined as the rollover estimate of volatility as defined in eq. (29) scaled by total assets of the year-quarter (Risk1) for mean reverting firms and the standard error of regression in equation (26) scaled by total assets for non-stationary firms. Market-to-book is the market value of equity plus long-term debt divided by total assets ((equity+dlttq)/atq). Tangible assets are property plant and equipment over total assets (ppentq/atq). Herfindahl index a particular industry is defined as the sum of squared market shares for all firms in a two-digit SIC industry in a year-quarter. The market share of a firm is defined as sales of that firm in a year-quarter divided by the total sales in the industry of firm in that year-quarter. LT profit to asset is rollover estimated long-term profit of a firm in a year-quarter as defined in eq. (28) divided by total assets of the corresponding year-quarter. Mean reversion speed is the rollover estimated mean reversion speed as defined in eq. (27) in a year-quarter scaled by total assets. The symbols *, **, and **** refer to estimates significantly different from zero at 10%, 5% and 1% confidence level, respectively. Heteroskedastic and autocorrelation robust standard errors clustered at firm level are provided in parenthesis.

Table 10. Factors affecting leverage at investment events with debt financing

	Mean reverting		Non-stationary	
	(1)		(2)	
Profitability	-1.396	***	-0.584	***
	(0.234)		(0.208)	
Risk	-0.129		2.887	**
	(0.354)		(1.289)	
Market-to-book	-0.174	***	-0.104	***
	(0.014)		(0.011)	
Profitability squared	-25.767		-28.313	***
	(5.120)		(4.628)	
Profitability squared x Market-to-book	12.925		8.995	***
	(2.782)		(1.749)	
LT profit to assets	1.354	***		
	(0.317)			
Mean reversion speed	0.036	***		
	(0.009)			
Controls:				
Size	-0.007	***	-0.008	**
	(0.003)		(0.003)	
Tangible	0.114	***	0.277	***
	(0.028)		(0.031)	
Herfindahl	-0.044		-0.248	***
	(0.030)		(0.039)	
Quarter FE	Yes		Yes	
Industry FE	Yes		Yes	
R^2	0.59		0.65	
Adj. R^2	0.54		0.62	
Firm obs.	518		409	
Total obs.	3,225		3,004	
F-test (Prob)	<0.0001		<0.0001	

Notes: The analysis is based on all non-financial, non-regulated firms with at least 40 consecutive earnings (oibdpq) which are mean reverting (see Table 7). All variables are defined in the note of Table 9. The analysis focuses on investment events defined as firm-quarter periods where a firm's year-to-date investment to assets ratio exceeds the 75% year-to-date quartile industry ratio and the long-term book debt ratio exceeds the 75% quartile industry ratio in that period. The symbols *, **, and **** refer to estimates significantly different from zero at 10%, 5% and 1% confidence level, respectively. Heteroskedastic and autocorrelation robust standard errors clustered at firm level are provided in parenthesis.

Table A1. Sensitivity analysis with respect to earnings volatility (σ)

Panel A: Optimal coupon and thresholds

σ	R_0	R_1	x_l	v_L	x_b
0.24	0.72	0.46	1.015	-0.658	-0.415
0.30	0.61	0.5	1.135	-1.028	-0.772
0.40	**0.52**	**0.55**	**1.341**	**-1.501**	**-1.191**
0.50	0.46	0.64	1.541	-1.856	-1.563

Panel B: Values at t = 0 and the investment trigger T

	Values at t = 0							Values at investment trigger T		
σ	Fb(x)	Db(x)	Ub(x)	NBb(x)	Lev_b	Inv_b	Cr_b	Lev_T	ΔLev	Cr_T
0.24	20.822	11.644	28.187	2.475	0.559	9.840	0.0018	0.616	0.057	0.0018
0.30	20.580	9.711	27.373	2.080	0.472	8.873	0.0028	0.553	0.081	0.0028
0.40	**20.556**	**7.997**	**26.720**	**1.692**	**0.389**	**7.856**	**0.0050**	**0.485**	**0.096**	**0.0052**
0.50	20.815	6.778	26.551	1.532	0.326	7.268	0.0079	0.450	0.124	0.0082

Notes: Initial earning level x = 1, risk-free rate r = 0.06, tax rate τ = 0.15, proportional bankruptcy costs b = 0.5, growth rate parameter e = 2, investment cost I = 10. For the mean-reverting stochastic model parameters we vary σ and use a mean reversion speed q = 0.1 and long-term mean of earnings θ = 1. ΔLev stands for change in leverage and is calculated as Lev_T - Lev_b. Base case parameters are highlighted in bold.

Table A2. Sensitivity with respect to priority rule: "me-first" priority for initial debt with sensitivity with respect to volatility

Panel A: Optimal coupon and thresholds

σ	R_0	R_1	x_I	v_L	x_b
0.23	0.68	0.51	1.006	-0.601	-0.471
0.30	0.58	0.54	1.163	-1.007	-0.825
0.40	**0.5**	**0.59**	**1.386**	**-1.461**	**-1.218**
0.50	0.45	0.67	1.599	-1.818	-1.565

Panel B: Values at t = 0 and the investment trigger T

			Values at t = 0					Values at investment trigger T		
σ	Fb(x)	Db(x)	Ub(x)	NBb(x)	Lev_b	Inv_b	Cr_b	Lev_T	ΔLev	Cr_T
0.23	20.888	11.166	28.273	2.551	0.535	9.935	0.0009	0.624	0.090	0.0016
0.30	20.585	9.417	27.161	2.068	0.457	8.644	0.0016	0.552	0.095	0.0029
0.40	**20.562**	**7.924**	**26.469**	**1.679**	**0.385**	**7.586**	**0.0031**	**0.487**	**0.102**	**0.0052**
0.50	20.823	6.899	26.290	1.521	0.331	6.989	0.0052	0.451	0.119	0.0082

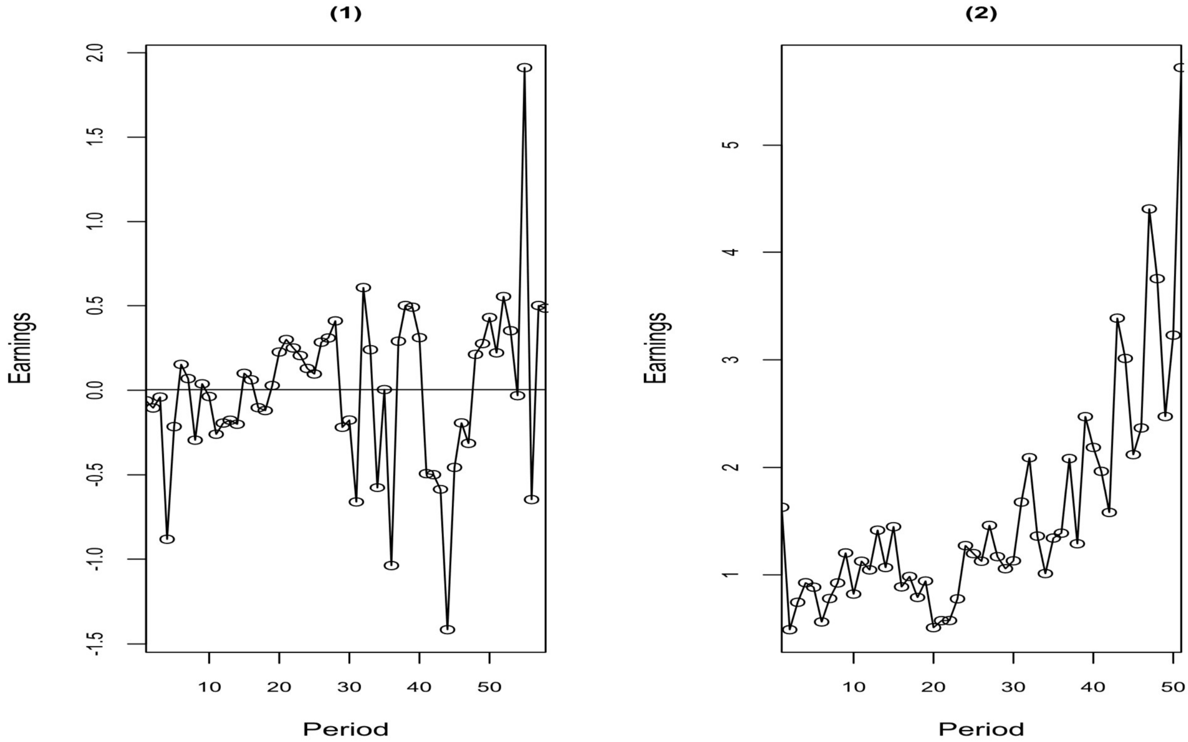
Notes: In the above sensitivity the following was used: initial earnings level $x = 1$, risk-free rate of r = 0.06, tax rate τ = 0.15 and proportional bankruptcy costs $b = 0.5$. For modelling the growth option, we use $e = 2$, investment cost $I = 10$. For the mean-reverting stochastic model parameters we vary σ, use a mean reversion speed q = 0.1 and long-term mean of earnings θ = 1. ΔLev stands for change in leverage and is calculated as Lev_T - Lev_b. Base case parameters highlighted in bold. In this sensitivity results we use "me-first" priority for first debt (see equations 10a and 10b).

Table A3. Sensitivity with respect to x for the model in the absence of growth option

x	x_L	x_A	R	Fb(x)	Db(x)	Lev_b
0.5	-0.990	-1.950	0.42	12.241	6.412	0.524
1	-0.843	-1.950	0.49	15.023	7.517	0.500
1.5	-0.700	-1.950	0.56	17.785	8.585	0.483
2	-0.601	-1.950	0.61	20.534	9.372	0.456
2.5	-0.504	-1.950	0.66	23.270	10.139	0.436
3	-0.409	-1.950	0.71	25.998	10.889	0.419
3.5	-0.335	-1.950	0.75	28.720	11.501	0.400
4	-0.262	-1.950	0.79	31.437	12.103	0.385
4.5	-0.190	-1.950	0.83	34.149	12.698	0.372
5	-0.120	-1.950	0.87	36.859	13.287	0.360
5.5	-0.051	-1.950	0.91	39.566	13.871	0.351
6.5	0.083	-1.950	0.99	44.974	15.025	0.334

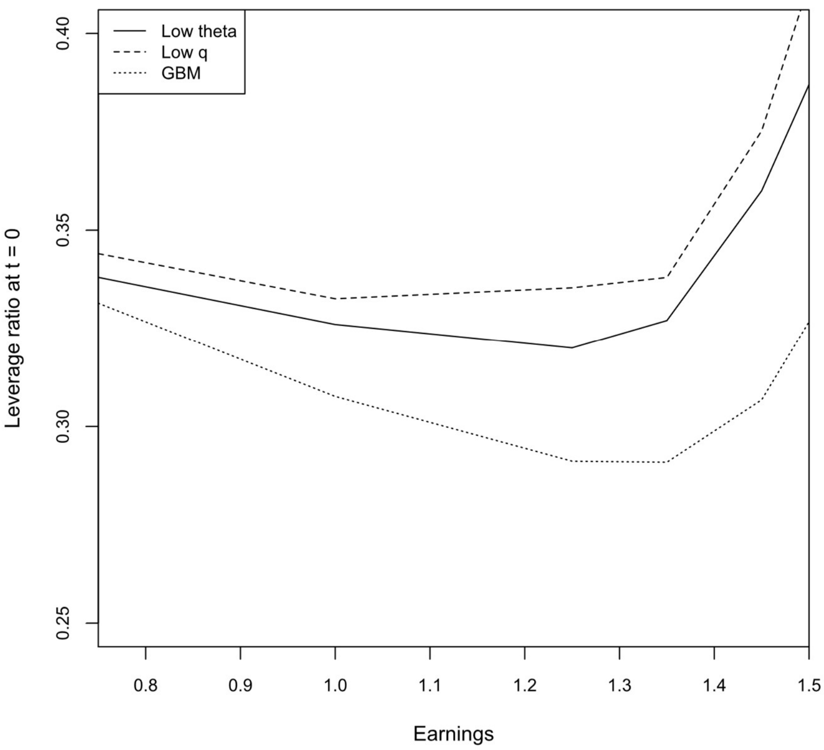
Notes: Risk-free rate r = 0.06, tax rate τ = 0.15, proportional bankruptcy costs b = 0.5. For the mean-reverting stochastic model parameters we use σ = 0.4, mean reversion speed q = 0.1 and long-term mean of earnings θ = 1. Lev_b is calculated as Db(x)/ Fb(x).

Figure 1. Examples of classifications of earnings processes



Notes: Plot 1 shows an example of a mean reverting process. It refers to firm “AM COMMUNICATIONS INC” with CUSIP number 001674100 with estimated mean reversion parameters θ = 0, q = 1.97 and σ = 0.69. Plot 2 shows an example of a firm found to be non-stationary (“ABS INDUSTRIES INC” with CUSIP = 000781104). Both plots show their earnings (oibdpq) unadjusted for seasonality for the whole periods of consecutive available data for each firm.

Figure A1. Robustness of U-shape of profitability with leverage in AMR mean reversion setting and also for non-stationary earnings (GBM)



Notes: For the mean reverting cases "low theta" and "low q" we use risk-free rate r = 0.06, tax rate $\tau = 0.15$, proportional bankruptcy costs $b = 0.5$, growth option rate $e = 2$, investment cost $I = 10$, $\sigma = 0.4$. For the "low theta" case we use mean reversion speed q = 0.1 and long-term mean of earnings $\theta = 0.75$. For the "low q" case we use q = 0.075 and long-term mean of earnings $\theta = 1$. For the GBM case we implement Hackbarth and Mauer (2012) with parameters (using the notation of our paper) r = 0.06, $\delta = 0.05$, tax rate $\tau = 0.15$, proportional bankruptcy costs $b = 0.5$, growth option rate $e = 1.6$, investment cost $I = 10$, $\sigma = 0.25$.

Appendix 1. Definition of variables of the theoretical model

$Eb(x)$ = Equity before investment (equity in stage 1).

$Fb(x)$ = Firm value before investment.

$Ub(x)$ = Value of unlevered assets before investment.

$TBb(x)$ = Tax benefits before investment.

$BCb(x)$ = Bankruptcy costs before investment.

$Db(x)$ = Debt before investment.

R_0= Coupon for $Db(x)$.

x_b = Bankruptcy threshold before investment.

x_I = Investment trigger

$Ea(x)$ = Equity after investment (equity in stage 2).

$Fa(x)$ = Firm value after investment.

$Ua(x)$ = Value of unlevered assets after Investment.

$TBa(x)$ = Tax Benefits after Investment.

$BCa(x)$ = Bankruptcy costs after Investment.

$Da_0(x)$ = Debt value of debt obtained at time zero after investment.

$Da_1(x)$ = Debt value of debt obtained at the investment trigger after investment.

R_1= Coupon for $Da_1(x)$.

x_L = Bankruptcy threshold following investment (in stage 2).

τ = Corporate tax rate

b = Proportional to unlevered assets bankruptcy costs

β_0 = share of initial debt holders at bankruptcy in stage 2 under equal priority.

$\beta_1 =$ share of second debt holders at bankruptcy in stage 2 under equal priority.

I = Investment cost

$R_T = R_0 + R_1$

$Lev_b = Db(x) / Fb(x)$: Leverage ratio at t = 0

$Cr_b = R_0 / Db(x) - r$: Credit spread of initial debt at t = 0

$Inv_b = I \cdot J(x)$ = Expected present value of investment costs

$NBb(x) = TBb(x) - BCb(x)$: Net benefits of debt

At the investment trigger:

$Lev_T = (Da_0(x) + Da_1(x)) / Fa(x)$: Total leverage ratio at the investment trigger

$\Delta Lev = Lev_T - Lev_b$: Change in leverage relative to initial stage

$Cr_b = R_0 / Da_0(x) - r$: Credit spread of initial debt at the investment trigger

$Cr_T = (R_0 + R_1) / (Da_0(x) + Da_1(x)) - r$: Credit spread of total debt at the investment trigger

Online Appendices

Appendix 1: Derivation of the homogeneous differential equation solution

Following standard replication arguments (example, Dixit and Pindyck, 1994, p.180) any contingent claim $P(x)$ on an underlying asset *x* that follows the mean reversion process defined in equation (1) should satisfy[17]:

$$T(P(x)) = \frac{1}{2}\sigma^2 P''(x) - q(x-\theta)P'(x) - rP(x) = 0, \quad x \in \Re \tag{A1}$$

To find the general solution of this homogeneous differential equation first set $\bar{\sigma} = \sigma/\sqrt{2q}$ and make the following change of variables:

$$z = \frac{x-\theta}{\bar{\sigma}}.$$

Then $P(x) = u(z)$, $P'(x) = \frac{1}{\bar{\sigma}} u'(z)$ and $P''(x) = \frac{1}{\bar{\sigma}^2} u''(z)$. Thus equation (A1) is transformed to:

$$q\, u''(z) - qz\, u'(z) - ru(z) = 0, \quad z \in \Re. \tag{A2}$$

Setting also $u(z) = w(z)e^{\frac{z^2}{4}}$, with $\nu = -\frac{r}{q} < 0$, deduce that $u'(z) = e^{\frac{z^2}{4}}\left(w'(z) + w(z)\frac{z}{2}\right)$ and $u''(z) = e^{\frac{z^2}{4}}\left(w''(z) + zw'(z) + w(z)\frac{1}{2}(1 + \frac{z^2}{2})\right)$. A simple calculation then shows that equation (A2) can be rewritten into:

$$w''(z) - \left[\frac{1}{4}z^2 - \left(\nu + \frac{1}{2}\right)\right] w(z) = 0, \quad z \in \Re. \tag{A3}$$

Equation (A3) is the real version of Weber's equation (Abramowitz and Stegun, 1972), that is:

$$w''(z) - \left[\frac{1}{4}z^2 + a\right] w(z) = 0, \quad z \in \mathbb{C}, \tag{A4}$$

where $a = -\nu - \frac{1}{2}$. The general solution of equation (A3) is given by:

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[17] To derive this general contingent claim differential equation, we assume risk-neutral investors and hence that the total required return on holding an asset in equilibrium is r = a(*x*)+δ where a(*x*)=q(θ-*x*) is the capital (gain) of asset *x* and δ the convenience yield. Thus, the implied convenience yield of holding the underlying asset *x* is δ = r-a(x). A similar approach is followed in Sarkar and Zapatero (2003).

$$w_g(z) = C_1 U(a,z) + C_2 U(a,-z). \quad \text{(A5)}$$

With C_1 and C_2 general constants and where:

$$U(a,z) = \frac{1}{2^{\xi}\sqrt{\pi}}\left[\cos(\xi\pi)\,\Gamma\left(\frac{1}{2}-\xi\right)\,y_1(a,z) - \sqrt{2}\sin(\xi\pi)\,\Gamma(1-\xi)\,y_2(a,z)\right] \quad \text{(A6)}$$

with

$$\xi = \frac{1}{2}a + \frac{1}{4},$$

$$y_1(a,z) = e^{-\frac{z^2}{4}}\ {}_1F_1\ \left(\frac{1}{2}a + \frac{1}{4};\ \frac{1}{2};\ \frac{z^2}{2}\right) \quad \text{(A7)}$$

and

$$y_2(a,z) = z\,e^{-\frac{z^2}{4}}\ {}_1F_1\ \left(\frac{1}{2}a + \frac{3}{4};\ \frac{3}{2};\ \frac{z^2}{2}\right) \quad \text{(A8)}$$

where ${}_1F_1\ (\alpha;\beta;z) = M(\alpha;\beta;z)$ is the confluent hypergeometric function (see Buchholz, 1969, Borodin and Salminen, 2002).

For the sake of being in the same line of notation with the literature concerning the limits of functions subsequently used we name $D_\nu(z) = U(a,z)$. Thus, focusing on the real solutions, the general solution (A5) can be re-written as:

$$w_g(z) = C_1 D_\nu(z) + C_2 D_\nu(-z) \qquad z \in \Re. \quad \text{(A9)}$$

Two useful asymptotic properties of the two linear independent solutions of equation (A3) (Abramowitz and Stegun (1972), equations 19.3.1 and 19.3.2, p.687, combined with equations 19.8.1 and 19.8.2, p.689) are the following:

$$\lim_{z\to\infty} e^{\frac{z^2}{4}}\,D_\nu(z) \;=\; \lim_{z\to\infty}\ z^{\nu}(1 + O(z^{-2}) = 0, \quad for\ \ \nu < 0 \quad \text{(A10)}$$

and

$$\lim_{z\to\infty} e^{\frac{z^2}{4}}\ D_\nu(-z) \sim \frac{\sqrt{2\pi}}{\Gamma(-\nu)}\ \lim_{z\to\infty}\ e^{\frac{z^2}{2}}\ z^{-\nu-1} = \infty. \quad \text{(A11)}$$

Since we have used the transformation $u(z) = w(z)e^{\frac{z^2}{4}}$ we can now move back to get the general solution of equation (A2) to be:

$$u_g(z) = C_1\, e^{\frac{z^2}{4}}\, D_\nu(z) + C_2\, e^{\frac{z^2}{4}}\, D_\nu(-z), \qquad z \in \Re.$$

Thus, we deduce that the solution of equation (A1) expressed in terms of *x* is given by:

$$P(x) = C_1\, e^{\frac{1}{4}\left(\frac{(x-\theta)\sqrt{2q}}{\sigma}\right)^2} D_\nu\left(\frac{(x-\theta)\sqrt{2q}}{\sigma}\right) + C_2 e^{\frac{1}{4}\left(\frac{(x-\theta)\sqrt{2q}}{\sigma}\right)^2} D_\nu\left(-\frac{(x-\theta)\sqrt{2q}}{\sigma}\right),\ x \in \Re. \qquad \text{(A12)}$$

For simplicity of presentation denote the general solution of (A1) as

$$P(x) = C_1 P_1(x) + C_2 P_2(x), \qquad \text{(A13a)}$$

with

$$P_1(x) = e^{\frac{1}{4}\left(\frac{(x-\theta)\sqrt{2q}}{\sigma}\right)^2} D_\nu\left(\frac{(x-\theta)\sqrt{2q}}{\sigma}\right),$$

and

$$P_2(x) = e^{\frac{1}{4}\left(\frac{(x-\theta)\sqrt{2q}}{\sigma}\right)^2} D_\nu\left(-\frac{(x-\theta)\sqrt{2q}}{\sigma}\right),$$

with equations (A10) and (A11) giving that:

$$\lim_{x\to\infty} P_1(x) = 0 \qquad \text{(A13b)}$$

$$\lim_{x\to-\infty} P_1(x) = \infty \qquad \text{(A13c)}$$

$$\lim_{x\to\infty} P_2(x) = \infty \qquad \text{(A13d)}$$

$$\lim_{x\to-\infty} P_2(x) = 0 \qquad \text{(A13e)}$$

Appendix 2: Derivation of solution for basic and general claims involving two boundaries

A2.1.Basic claim paying one dollar at v_L after investment

Consider the following differential equation problem:

$$T^*\big(Q(v)\big) = 0, \quad v \in \Re \qquad \text{(A14)}$$
$$\lim_{v\to\infty} Q(v) = 0$$
$$Q(v_L) = 1$$

where $T^*(\theta^*, \sigma^*) \equiv T(e\theta, e\sigma)$. The solution for $Q(v)$ is given by applying (A13a):

$$Q(v) = C_1P_1(v) + C_2P_2(v)$$

Applying the first boundary condition in (A14) combined with equation (A13d) gives $C_2 = 0$. Then the second boundary condition gives $C_1 = \frac{1}{P_1(v_L)}$. Thus, the solution for this basic claim paying one dollar at v_L after investment is:

$$Q(v) = \frac{P_1(v)}{P_1(v_L)} \qquad \text{(A15)}$$

A2.2. Basic claims for homogeneous equations before investment

$J(x)$ and $L(x)$ are basic claims where $J(x)$ pays one dollar at x_I and zero when x_b is reached and $L(x)$ pays one dollar at x_b and zero when x_I is reached.

A. Derivation of $J(x)$

Consider the following differential equation problem:

$$T\big(J(x)\big) = 0, \quad x \in \Re \qquad \text{(A16)}$$
$$J(x_I) = 1$$
$$J(x_b) = 0$$

The solution $J(x)$ satisfies (A13) hence:

$$J(x) = C_1P_1(x) + C_2P_2(x)$$

Applying the boundary conditions in (A16) results in:

$$C_1 = \frac{P_2(x_b)}{D(x_I, x_b)}, \qquad C_2 = -\frac{P_1(x_b)}{D(x_I, x_b)},$$

where

$$D(x_I, x_b) = P_1(x_I)P_2(x_b) - P_1(x_b)P_2(x_I).$$

Thus, the solution for $J(x)$ is:

$$J(x) = \frac{P_2(x_b)}{D(x_I,x_b)} P_1(x) - \frac{P_1(x_b)}{D(x_I,x_b)} P_2(x). \qquad \text{(A17)}$$

B. Derivation of $L(x)$

Consider now the corresponding problem for $L(x)$ which is given by:

$$T\big(L(x)\big) = 0, \quad x \in \Re \qquad \text{(A18)}$$

$$L(x_I) = 0$$

$$L(x_b) = 1$$

Applying the boundary conditions results in the following solutions for the constants:

$$C_1 = -\frac{P_2(x_I)}{D(x_I, x_b)}, \qquad C_2 = \frac{P_1(x_I)}{D(x_I, x_b)} .$$

Thus, the solution for $L(x)$ is:

$$L(x) = -\frac{P_2(x_I)}{D(x_I,x_b)} P_1(x) + \frac{P_1(x_I)}{D(x_I,x_b)} P_2(x) \qquad \text{(A19)}$$

A2.3. Basic claims for linear homogeneous equations

Consider now the following problem regarding a contingent claim $N(x)$:

$$T\big(N(x)\big) = 0, \quad x \in \Re \qquad \text{(A20)}$$

$$N(x_I) = A$$

$$N(x_b) = B$$

It can be easily shown that the solution of problem (A20) can be written in terms of the basic claims $J(x)$ and $L(x)$ in the following way:

$$N(x) = A\ J(x) + B\ L(x). \qquad \text{(A21)}$$

A2.4. Basic claims for non-homogeneous equations

Consider now a more general contingent claim $M(x)$ which may pay $g(x)$ expressed by:

$$T\big(M(x)\big) + g(x) = 0, \quad x \in \Re \qquad \text{(A22)}$$
$$M(x_I) = A$$
$$M(x_b) = B$$

Since $T(\cdot)$ is a linear differential operator then the general solution is given by the expression:

$$M(x) = M_h(x) + M_p(x), \qquad \text{(A23)}$$

where $M_h(x)$ is a solution of a corresponding homogeneous problem

$$T\big(M_h(x)\big) = 0$$

(that is $g(x) = 0$) and $M_p(x)$ is one solution of problem (A20). To find which boundary conditions $M_h(x)$ should satisfy notice that:

$$M_h(x_I) = M(x_I) - M_p(x_I) = A - M_p(x_I)$$

$$M_h(x_b) = M(x_b) - M_p(x_b) = B - M_p(x_b)$$

The problem for $M_h(x)$ is in the form of problem (A20) and its solution is given by equation (A21). Thus, we obtain the solution:

$$M_h(x) = \Big(A - M_p(x_I)\Big)J(x) + \Big(B - M_p(x_b)\Big)L(x).$$

As a result, the solution for the value of $M(x)$ is:

$$M(x) = \Big(A - M_p(x_I)\Big)J(x) + \Big(B - M_p(x_b)\Big)L(x) + M_p(x). \qquad \text{(A24)}$$

Equation (A24) is general enough to value securities (equity, debt) and firm value prior to investment depending on the payment $g(x)$ (which define $M_p(x)$ for the particular claim) and the boundary values A and B. Note that for debt holders $g(\cdot)$ is not a function of x.

Appendix 3: Detailed proofs of security and firm valuation solutions

A3.1. General solution of the problem

Consider the differential equation of the form:

$$T\big(y(x)\big) + ax + b = 0, \quad x \in \Re. \tag{A25}$$

The general solution of this problem is given by $y_g(x) = y_h(x) + y_p(x)$, where $y_h(x)$ is a solution of $T\big(y(x)\big) = 0$ and $y_p(x)$ (particular solution) is one solution of equation (A25). From equation (A13a) we have that $y_h(x) = C_1P_1(x) + C_2P_2(x)$. For the particular solution consider that $y_p(x) = k_1x + k_2$. Then $y_p'(x) = k_1$ and $y_p''(x) = 0$. Plugging in equation (A25) where $T(\cdot)$ is given by equation (A1) we get:

$$-q(x-\theta)k_1 - r(k_1x + k_2) + ax + b = 0.$$

Rearranging the terms, one gets:

$$(-(q+r)k_1 + a)x + q\theta k_1 - rk_2 + b = 0.$$

This gives that:

$$k_1 = \frac{a}{q+r}$$

and

$$k_2 = \frac{1}{r}\left(\frac{q\vartheta a}{q+r} + b\right)$$

Thus, the general solution of equation (A25) is given by:

$$y_g(x) = C_1P_1(x) + C_2P_2(x) + \frac{a}{q+r}x + \frac{1}{r}\left(\frac{q\vartheta a}{q+r} + b\right) \tag{A26}$$

A3.2. Values after investment

A3.2.1. Value of unlevered assets

The value of unlevered assets after investment satisfies the following differential equation:

$$T^*\big(Ua(v)\big) + v(1-\tau) = 0, \quad v \in \Re. \tag{A27}$$

The general solution of equation A27 is given by equation (A26) with $a = 1 - \tau$ and $= 0$:

$$Ua(v) = C_1P_1(v) + C_2P_2(v) + \left[\frac{1}{q+r}v + \frac{q\theta^*}{r(q+r)}\right](1-\tau) \tag{A28}$$

The value of unlevered assets must also satisfy the following boundary conditions:

$$\lim_{v \to \pm\infty} Ua(v) = Ua_p(v) \tag{A28}$$

Equation (A13b) then suggests that $C_2 = 0$ and equation (A13c) suggests that $C_1 = 0$. Thus

$$Ua(v) = \left[\frac{1}{q+r} v + \frac{q\theta^*}{r(q+r)}\right](1-\tau) \tag{A29}$$

$Ua(v)$ can turn negative for sufficiently negative v. The value of v_A at which the value of unlevered assets is zero is the solution of $Ua(v_A) = 0$ which suggests that $v_A = -\frac{q\theta^*}{r}$. Since the value of unlevered assets is obtained (net of bankruptcy costs) by debt holders when the firm goes bankrupt at optimally determined v_L we need to ensure that if $v_L < v_A$ debt holders do not obtain a negative value and thus if $v_L < v_A$, $Ua(v_L)$ is set to zero.

A3.2.2. Equity value

Equity value after investment satisfies the following differential equation:

$$T^*\big(Ea(v)\big) + (v - R_0 - R_1)(1-\tau) = 0, \quad v \in \Re \tag{A30}$$

The general solution of equation A30 is given by equation (A26) with $a = 1 - \tau$ and $b = -(1 - \tau)(R_0 + R_1)$:

$$Ea(v) = C_1 P_1(v) + C_2 P_2(v) + \left(\frac{1}{q+r} v + \frac{q\theta^*}{r(q+r)} - \frac{R_0+R_1}{r}\right)(1-\tau). \tag{A31}$$

Equity must also satisfy

$$\lim_{v \to \infty} Ea(v) = Ea_p(v) \tag{A32}$$

and

$$Ea(v_L) = 0 \tag{A33}$$

Equation (A13d) suggests that $C_2 = 0$ and by (A33) we obtain that

$$C_1 = -\frac{Ea_p(v_L)}{P_1(v_L)}$$

Thus, we obtain that:

$$Ea(v) = Ea_p(v) - Ea_p(v_L)\frac{P_1(v)}{P_1(v_L)} \qquad \text{(A34)}$$

Setting $v = ex$ define

$$\tilde{E}a(x) = Ea(ex) = Ea_p(ex) - Ea_p(v_L)\frac{P_1(ex)}{P_1(v_L)} \qquad \text{(A35)}$$

A3.2.3. Debt values

Debt value after investment for the initial debt issued at time zero $Da_0(v)$ and the second debt issued at the investment trigger $Da_1(v)$ satisfy the following:

$$T^*\big(Da_i(v)\big) + R_i = 0, \qquad i = 0,1 \qquad v \in \Re \qquad \text{(A36)}$$

The general solution of equation (A36) is given by equation (A26) with $a = 0$ and $b = R_i$:

$$Da_i(v) = C_1P_1(v) + C_2P_2(v) + \frac{R_i}{r}$$

(A37)

Debt must also satisfy two boundary conditions. The first one is given by:

$$\lim_{v\to\infty} Da_i(v) = \frac{R_i}{r} \qquad \text{(A38)}$$

The second boundary depends on the priority structure. Under equal priority:

$$Da_i(v_L) = \beta_i\,(1-b)\,Ua(v_L) \qquad \text{(A39)}$$

In the case the first creditors have secured priority to other creditors then the boundary conditions become:

$$Da_0(v_L) = min\,\left|(1-b)\,Ua(v_L), \frac{R_0}{r}\right| \qquad \text{(A40)}$$
$$Da_1(v_L) = (1-b)\,Ua(v_L) - Da_0(v_L))$$

In the case that second debt holders have secured priority to other creditors then the boundary conditions become:

$$Da_1(v_L) = min\ \left[(1-b)\ Ua(v_L), \frac{R_1}{r}\right] \quad \text{(A41)}$$

$$Da_0(v_L) = (1-b)\ Ua(v_L) - Da_1(v_L)$$

Equation (A36) combined with (A13d) suggests that $C_2 = 0$. Thus $Da_i(v) = C_1 P_1(v) + \frac{R_i}{r}$. Depending on priority structure, applying boundary conditions (A39), (A40) or (A41) deduce that:

$$C_1 = \frac{Da_i(v_L) - \frac{R_i}{r}}{P_1(v_L)}$$

and thus

$$Da_i(v) = \frac{R_i}{r} + \left(Da_i(v_L) - \frac{R_i}{r}\right)\left(\frac{P_1(v)}{P_1(v_L)}\right) \quad \text{(A42)}$$

Setting $v = ex$ define

$$\widetilde{D}a_i(x) = Da_i(ex) = \frac{R_i}{r} + \left(Da_i(ex_L) - \frac{R_i}{r}\right)\left(\frac{P_1(ex)}{P_1(ex_L)}\right) \quad \text{(A43)}$$

A3.3. Values before investment

A3.3.1 Value Unlevered before investment

Following similar arguments as the ones used to derive the value of unlevered assets after investment one can show that the value of unlevered assets before investment $Ub(x)$ is given by:

$$Ub(x) = \left[\frac{1}{q+r}x + \frac{q\theta}{r(q+r)}\right](1-\tau) \quad \text{(A44)}$$

To avoid negative liquidation values for initial debt holders at bankruptcy if $x_B < x_A$ then $Ub(x_B) = 0$ where $x_A = -\frac{q\theta}{r}$ is the threshold where $Ub(x)$ becomes zero.

A3.3.2. Debt value before investment

Debt $Db(x)$ satisfies the following differential equation:

$$T\big(Db(x)\big) + R_0 = 0, \qquad x \in \Re \tag{A45}$$

The general solution of equation (A43) is given by equation (A26) with $a = 0$ and $b = R_0$:

$$Db(x) = Db_h(x) + \tfrac{R_0}{r} \tag{A46}$$

Debt before investment must also satisfy the following boundary conditions:

$$Db(x_I) = Da_0(x_I)$$

$$Db(x_b) = (1-b)\,Ub(x_b).$$

Equation (A24) then suggests that the solution of the problem is given by:

$$Db(x) = \left(Da_0(x_I) - \tfrac{R_0}{r}\right)J(x) + \left((1-b)\,Ub(x_b) - \tfrac{R_0}{r}\right)L(x) + \tfrac{R_0}{r} \tag{A47}$$

A3.3.3 Equity and firm value before investment

The equity function before investment satisfies the following differential equation:

$$T\big(Eb(x)\big) + (x - R_0)(1-\tau) = 0, \quad x \in \Re \tag{A48}$$

The general solution is given by equation (A26) with $a = 1 - \tau$ and $b = -(1-\tau)R_0$:

$$Eb(x) = Eb_h(x) \;+\; Eb_p(x) = C_1P_1(x) + C_2P_2(x) + \left(\frac{1}{q+r}x + \frac{q\theta}{r(q+r)} - \frac{R_0}{r}\right)(1-\tau)$$

Equity should also satisfy the following boundary conditions:

$$Eb(x_I) = Ea(v_I) - I + Da_1(v_I)$$

$$Eb(x_b) = 0$$

Equation (A24) then implies that solution of the problem is:

$$Eb(x) = \big(Ea(v_I) - I + Da_1(v_I) - Eb_p(x_I)\big)J(x) - Eb_p(x_b)\,L(x) + Eb_p(x) \tag{A49}$$

Note that with $v_I = ex_I$ this becomes:

$$Eb(x) = \big(Ea(e\,x_I) - I + Da_1(e\,x_I) - Eb_p(x_I)\big)J(x) - Eb_p(x_b)\,L(x) + Eb_p(x)$$

Firm value before investment is then given by the sum of equity plus debt after investment:

$$Fb(x) = Eb(x) + Db(x) \tag{A50}$$

Appendix 4: Additional sensitivity results

A.4.1. Additional sensitivity results of the main model with AMR

Table A1 provides sensitivity results with respect to the volatility of earnings σ of the main model with mean reversion described in section 2. In Panel A, consistent with a real options explanation we observe that an increase in volatility results in a delay of the option to invest (x_I increases) and a delay in default decisions (v_L and x_b thresholds decrease). In Panel B we observe that a higher earnings volatility has a U-shape effect on firm value (Fb(x)), decreases the value of unlevered assets (Ub(x)), results in a lower leverage ratio at t = 0 (Lev_b), lower net benefits of debt (NBb(x)) and decreases the expected present value of investment costs (Inv_b). The latter effect implies that investment is less likely to occur. As expected, leverage decreases with volatility (see also Sarkar and Zapatero, 2003). Despite the decrease in leverage, credit spreads increase with σ. Higher volatility also reduces leverage ratios and has a positive impact on credit spreads at the investment trigger. With respect to leverage dynamics, we find when firms reach a stage of exercising their investment with no further options available leverage exhibits an increase relative to prior levels. Intuitively, at higher volatility, investment is triggered at a higher revenue level, which enables the firm to move to higher levels of leverage.

We have also explored whether the above sensitivity results change when long-term profitability is different (results not shown for brevity). We find that when long-term profitability is high an increase in volatility reduces firm value. This is intuitive since higher volatility increases the likelihood of moving away from highly valuable prospects. The opposite result is obtained when long-term profitability is low in which case firm value becomes strictly increasing with volatility. This explains the U-shape observed in our base case (average) long-term profitability. Despite these differences with our base case, all other results remain when volatility is higher: the investment and default are delayed, leverage decreases, credit spreads increase and changes in leverage at the investment trigger relative to previous levels increase.

[Insert Table A1 here]

Next we examine sensitivity results with respect to alternative priority rules of debt at default. Here, we focus only on the case of "me-first" priority for initial debt (see equations 10a and 10b) and contrast it with the results of equal priority used in our earlier analysis. Table A2 shows sensitivity with respect to the volatility of earnings using "me-first" priority. These results can be contrasted with those of Table 1 in the main text where a similar sensitivity was conducted with equal priority for debt holders.[18]

[Insert Table A2 here]

Compared to the case of equal priority (see Table 1, panel B) we observe a slight increase in firm value under a "me-first" priority rule for initial debt. This is like the result of Hackbarth and Mauer (2012) where they find that "me-first" results in higher values.[19] The differences in firm value however under different priority rules is small. We observe a more significant conservatism of debt raised at $t = 0$ which can be seen by the lower initial coupon R_0 and initial debt level raised at $t = 0$ under the "me-first" for initial debt compared to the case of equal priority. Instead, the firm under a "me-first" for initial debt priority rule preserves more financial flexibility to issue more debt when the investment is exercised. Indeed, R_1 at the investment trigger is higher under "me-first" compared to the case of equal priority. Despite the increase in the coupon of new debt, at the investment trigger the leverage ratios and overall credit spreads remain similar between the two priority rule cases. This is because there is a counterbalancing effect caused by lower initial leverage under "me-first" balancing out the higher subsequent R_1 at the investment trigger. The initial conservatism in debt levels combined with higher protection for initial debt and more delayed default (see Panel A compared to the case of Table 1) results in a substantial decrease of initial ($t = 0$) credit spreads (relative to the equal priority rule of Table 1). With respect to other firm policies (see Panel A), we observe that under a "me-first" for initial debt priority rule the firm delays investment more (see also the lower expected investment cost incurred in Panel B). This underinvestment effect occurring under "me-first" confirms the results of Hackbarth and Mauer (2012) (see p.774) within a framework of mean reversion in earnings. We finally observe that the

[18] We have conducted extensive sensitivity across all parameters using a "me-first" priority rule for initial debt. Like Hackbarth and Mauer (2012) we find no significant differences in firm values or leverage ratios. Importantly, the predictions highlighted in the rest of the paper appear intact under a "me-first" priority rule. Thus, in the main text we discuss only the qualitative implications of the "me-first" compared to the equal priority rule.

[19] Hackbarth and Mauer (2012) also point out that a "me-first" priority rule results in solutions for firm value which is closer among all rules to the (ideal) optimal priority rule.

directional effects of volatility remain the same as in the case of equal priority. We note however the more pronounced increase in the leverage ratio at the last stage of investment (when no more growth options are available) relative to the initial leverage ratio under a "me-first" for initial debt compared to the equal priority case.

A.4.2. Additional sensitivity results showing the U-shape of leverage with x

We have investigated alternative parametrizations of the model and have found that the U-shape of leverage with profitability x is robust. Below we illustratively show that the U-shape holds also for low long-term profitability and when q is small. The latter case approximates the effect of profitability on leverage that one would expect for non-stationary firms $q \to 0$. To confirm that the U-shape holds for non-stationary firms we have also implemented the Hackbarth and Mauer (2012) model with earnings following a GBM. The figure also shows that the U-shape exists for the case where the earnings process follows a GBM like in the model of Hackbarth and Mauer (2012). We have conducted several other sensitivity results within the Hackbarth and Mauer (2012) model showing that the U-shape applies for alternative parametrizations (results available upon request). We find that the U-shape is less pronounced when the expansion factor of the option is high but still exists at high values of x.

[Insert Figure A1]

A.4.3. Single-stage model results showing a negative relation between leverage and x when x follows AMR

To investigate the drivers of the U-shape we have run sensitivity results of the model assuming the absence of the growth option. This model is described in Sarkar and Zapatero (2003), however here we use AMR instead of GMR assumption. Our sensitivity results show (see below) show that the negative relation between leverage and x is retained at high x values. Overall, these results indicate that the presence of the growth option is the driver of the U-shape.

[Insert Table A3 here]